EARTHWORK HAUL AND OVERHAUL

INCLUDING

ECONOMIC DISTRIBUTION

BY

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PREFACE

This book presents answers to all questions on computation of overhaul and on the use of the mass curve in planning distribution, which have arisen during an experience covering a wide range of conditions.

Part I is planned to serve five classes of men: railroad engineers, railroad contractors, computers, students, and teachers.

Engineers responsible for overhaul computation may be aided in selecting a method of computation by reading Sections 47–55, 146 and 203; and can obtain uniform results by merely directing their subordinates to follow the instructions given in Chapter VI or Chapter VII for the selected method.

Computers, having been directed to use a given method of computing overhaul, will find in either Chapter VI or Chapter VII a complete plan of procedure under the method.

Railroad contractors about to submit bids on work involving overhaul, will find the descriptions of bases and methods of computation of overhaul, given in Chapters V, VI, and VII, a ready means of coming to a definite understanding with railroad engineers as to the way in which the overhaul is to be computed.

Students of overhaul, in school or out, will find in Chapters I–V a full presentation of each of the elements of overhaul computation. The beginner is advised to proceed at once to compute the overhaul for some simple case similar to that shown in Fig. 19, by Method IV (Sections 89–99), studying the references under each step no more than may be necessary to execute the step intelligently. A working knowledge of the essentials and routine of one method thus acquired furnishes the groundwork necessary for the most profitable study of other portions of the book.

Men called upon from time to time, in class-room or office, to give instruction in overhaul computation, will find Part I a treatise from which portions can be selected to fulfill requirements for a course short or long according to the time available for the subject. The fact that the writer is entitled to full membership in this class may be taken as a guaranty that the teacher's interests have not been overlooked herein.

Part II is devoted to the elements of the problem of economic distribution, and presents a thorough treatment of the solution of this problem by the use of the mass curve.

The attention of those who are familiar with the previously-printed matter bearing on haul and overhaul is directed to Sections 2–9 (swell and distribution); 15–19 (concerning the mass curve); 43, 44 (crosshaul); Chapter VI (data and solution of a simple overhaul problem, by each of eight methods); Chapter VII (data and solution of complex overhaul problem from practice, by each of five methods); Fig. 36 (statement of overhaul computed by various methods); and nearly all of Part II.

The writer's indebtedness to the published matter on overhaul, especially to the Proceedings, 1906, of the American Railway Engineering Association, will be apparent to the well-informed. The drawings, with one or two exceptions, have been prepared by Messrs. N. M. Halcombe and A. C. Sandstrom, at present studying civil engineering in Stanford University. Mr. Sandstrom has taken a lively interest in the text as well; and his criticisms and suggestions at many points have caused desirable changes in and additions to the original manuscript.

The writer will be glad to receive suggestions for the improvement of any part of the book, from those to whom it does not prove entirely satisfactory.

J. C. L. Fish.

Stanford University, California
May 20, 1912.

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This chapter sets forth the principles of construction and interpretation of the mass curve; describes the horizontal and the oblique balancing line; and discusses the effect of drafting-errors on results obtained from the mass curve.

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This chapter describes how, by eye, by arithmetic, and by mass curve, to determine the limits of a body of material in cut or in fill from given conditions; and how to determine the center of volume or mass of such body. (Figs. 19, 21, 21a, and 24 face p. 96.)

CHAPTER IV

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In this chapter are described eight methods (so-called) of finding the center of gravity of a body of material, ranging from the roughly approximate to the practically exact, and using the eye, arithmetic, or the mass curve. All the methods are applied to the same problem, namely, finding the center of gravity of the cut between 9 + 00 and 12 + 28 of Fig. 19 (facing p. 96).

CHAPTER V

Overhaul, Free Haul, and Crosshaul

In this chapter overhaul is defined; a digest of American practice in computing overhaul is given; the American Railway Engineering Association's specification, which is used as the basis for all the problems worked in this book, is presented; and three methods — by eye, by arithmetic, and by mass curve — of determining free-haul limits are explained. Crosshaul is discussed; and eight methods of computing overhaul are characterized. (Fig. 19 faces p. 96; Fig. 36, p. 67; Fig. 38, p. 120.)

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being assumed to lie at its center of length. The limits are computed by arithmetic.)

Overhaul of Fig. 19 computed by Method VII

(In Method VII the center of gravity of each body of cut or fill is computed, by means of the mass curve, by the method of moments. All limits are determined by the mass curve.)

Overhaul of Fig. 19 computed by Method VIII

(In Method VIII the center of gravity of each cut or fill is found by the method of moments, using arithmetic, the center of gravity of each station-volume being found by computing its distance from the center of length. All limits are computed by arithmetic.)

CHAPTER VII

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(In Method V center of gravity of body of cut or fill is assumed to lie at its center of volume, and is determined by mass curve. Limits are determined by mass curve.)

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(In Method VI the center of gravity of a body of cut or fill is computed, arithmetically, by the method of moments, the center of gravity of each station-volume being assumed to lie at its center of length. The limits are computed by arithmetic.)

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PART II

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Economic Balancing Line for Mass Curve Plotted from Cut-Volumes and Equated Fill-Volumes, or from Cut-Volumes and Fill-Volumes when Swell-Factor is Unity

This chapter discusses the application of the principle of economic distribution to each of several typical forms of mass curve, progressing from the simple to the complex. All the mass curves of this chapter are assumed to be plotted from pay yards, that is, from yards in place. All the mass curves of the following chapter are assumed to be plotted from yards of swelled material. Otherwise the two chapters are similar. (Figs. 50–60a face p. 146.)

CHAPTER X

Economic Balancing Line for Mass Curve Plotted from Fill-Volumes and Equated Cut-Volumes

This chapter discusses the application of the principle of economic distribution to each of several typical forms of mass curve, progressing from the simple to the complex. The mass curves here represent cubic yards of swelled material, while the mass curves of the preceding chapter represent cubic yards in place.

EARTHWORK HAUL AND OVERHAUL

PART I

CHAPTER I

CONSIDERATIONS PRELIMINARY TO COMPUTATION OF HAUL

This chapter shows the necessity of adopting an ideal distribution, instead of the actual, as a basis of computing haul or overhaul; that, under some common conditions, any prediction of swell of material must be subject to considerable uncertainty; and gives a method for determining the most reasonable swell factors in those cases where field measurements for exact determination are not available.

1. **Haul defined.** — Suppose that the material between stations *m* and *n* (Fig. 1) just makes, or is assumed to make (Sec. 2), the

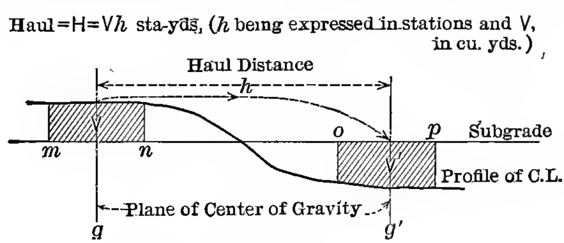


FIG. 1.

fill between stations *o* and *p*. The "haul" which results from moving the material of cut *mn* into fill *op* is equal to the volume of the cut *mn* multiplied by the distance between the center of gravity of cut *mn* and the center of gravity of fill *op*. Thus haul is expressed in compound units. In railroad work the haul distance is usually expressed in stations of 100 ft., and the volume in cubic yards. Hence the customary unit of haul is the station-yard (hereafter written sta-yd.): the haul resulting from hauling 1 c.y. of material a distance of 1 sta. So 50 sta-yds. is the result of hauling 1 c.y. 50 stas.; 50 c.y. 1 sta.; or 12.5 c.y. 4 stas.; and so on.

Let V = volume (c.y.) in the cut *mn*.
V' = volume (c.y.) in the fill *op*.
g = station and plus of the center of gravity of the cut *mn*.
g' = station and plus of the center of gravity of the fill *op*.
$h = g' - g$ = haul distance = distance (stations) between the center of gravity of the fill and the center of gravity of the cut.
H = the haul (sta-yds).

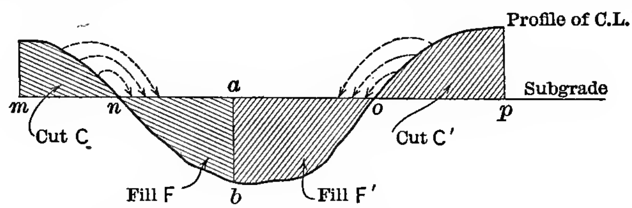
Then

$$H = Vh \quad . \; . \; . \quad (\text{not } V'h, \text{ unless } V' = V). \tag{1}$$

So, then, to ascertain the haul resulting from transporting a given body of material from a given position in a cut to a given position in a fill, we must (1) ascertain the volume of the given body of material "in place" (that is, in its original position in the cut); and (2) find the position of the center of gravity of that body of material in place and the position of the center of gravity of the same body of material after it is hauled to the fill. The reader is supposed to be familiar with earthwork measurements and volume computation. For reasons which will appear later, methods of determining center of gravity are deferred, to be taken up after attention has been given to the distribution and swell of material, and to the mass curve.

2. Disposition of material, actual and assumed; simple case. — In some cart-work the cut and fill, starting at the grade point, are carried forward in opposite directions simultaneously and with full cross-section, as shown in Figs. 2 and 12. Successive slabs across the cut are broken down, and hauled and dumped over the end of the fill where the material of each slab forms another slab on the end face of the fill. Under such working conditions the ends of the two fills, F and F', will approach each other as the work progresses and will meet in some plane of cross-section, as ab. In all cases like this the position of the center of gravity of each body of material, whether in place or in fill, is readily determined.

Let us consider, now, that the fill no (Fig. 2) is made in the following manner: (1) A trestle is built across the depression to be filled. (2) Material is hauled from cut C and dumped from



"IDEAL" CART CUTS AND FILLS

FIG. 2.

the trestle, making a narrow fill — a trestle fill — from n to o. (3) Additional material is taken from cut C and dumped over the side of the trestle fill, widening the fill irregularly. (4) Material from the other cut, C', is dumped over the sides of the partially widened fill, until the fill is completed to the desired dimensions. In this case, which is typical of fills made with cars, the material from the two cuts is intermingled in such manner that it is impracticable to measure the dimensions of the body of material in fill which has come from either cut. Hence it is impracticable to determine the position of the center of gravity of that material which comes from each cut. When figuring haul for a fill made under the conditions just described, the only practicable procedure is to assume, for this purpose, that the material hauled from cut C to fill no was so placed as to form a continuous full-sectioned portion of the fill, meeting a similar portion built from o with material hauled from cut C'.

In short, in computing haul we deal with the fill as if it were an ideal cart-fill. The position of the ideal, separating transverse plane is found by the method given in Sec. 20.

Again, let us consider the case of a cut made in the following manner (Fig. 3). A steam-shovel starting near *n*, makes a cutting to the vicinity of *o*, and then backs up to *n*. This process is repeated until all the material is removed from cut *no*. The steam-shovel is served by two trains. Part of the time both trains haul material to one fill, part of the time to the other, and still a third part of the time one train hauls to fill *mn* and the other train to fill *op*, according to the varying conditions on the

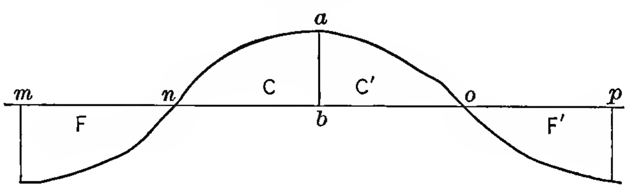
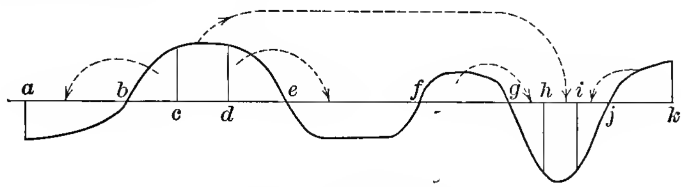


FIG. 3.

work. In this case it is evidently impracticable to find the position of the center of gravity of the space originally occupied by the material hauled to the left, or to find the position of the center of gravity of the space originally occupied by the material hauled to the right. The only practicable method of computing the haul in cases here typified is to substitute for this purpose an ideal cart-cut for the steam-shovel cut; that is, we must assume that a transverse plane, as *ab*, separates that portion of the cut from which material was hauled to the right, from that portion from which the material was hauled to the left.

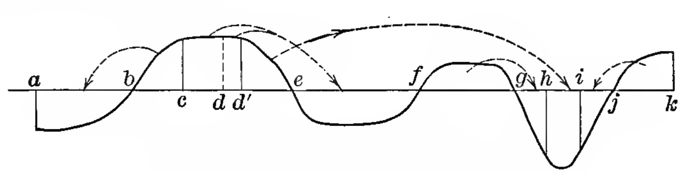
3. Disposition of material: actual and assumed; complex case. — Fig. 4 shows by arrows the ideal distribution of the material, assuming that the maximum haul does not exceed the distance of profitable haul (Secs. 206, 207). Fill *ab* is made from the adjacent material *bc*. There is more than enough material in *ce* to make the fill *ef*, so fill *ef* is made from *de*, leaving a surplus of material, *cd*. Fill *gh* takes all the material from cut *fg*. The surplus *cd* is hauled to *hi* and the remainder *ij* of fill *gj* is made from *jk*.



"IDEAL" DISTRIBUTION
OF
MATERIAL BY CARTS—NO CROSS HAUL

FIG. 4.

If the work be done by carts, the distribution of the material will normally be the ideal indicated. The order in which the parts of the work are attacked is of no significance. We might start a cart gang at *b*, a second at *e*, a third at *g*, and a fourth at *j*, simultaneously, and put a gang on *d* after the roadbed was completed between *d* and *h*; or, we might have only one gang to take up the cuts in any order, except that the cut *cd* would not ordinarily be taken out till the roadbed from *d* to *h* had been completed, and so on.



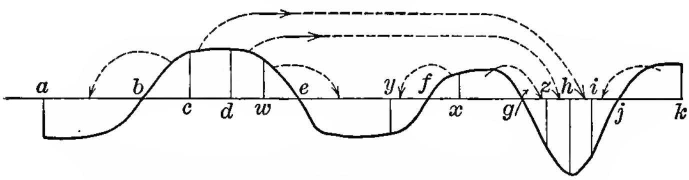
ANOTHER DISTRIBUTION—NO CROSS HAUL

FIG. 5.

The disposition indicated above is called ideal because there is no crosshaul (Sec. 43) and the haul (Sec. 1) is a minimum.

The profile of Fig. 4 is repeated in Figs. 5 and 6. Fig. 5 shows a little variation from Fig. 4. It is plain that the two figures indicate the same amount of haul (sta-yds.). However Fig. 4, and not 5, shows the normal disposition for carts.

In Fig. 6 there is a surplus of material at the left and a deficit at the right. Suppose *fx* hauled to *fy*, causing a surplus at *dw*, which results in hauling *dw* to *zh*. Fig. 6 indicates more haul (sta-yds.) than does Fig. 4, for in Fig. 6 we have crosshaul between *y* and *x*.



DISTRIBUTION SHOWING CROSS HAUL AT *x y*

FIG. 6.

Now if the work be done by contract, and the contractor be directed to dispose of the material as indicated by arrows in Fig. 6, the haul should be calculated on the basis of the distribution therein indicated by the arrows; however, if the contractor, having been directed to dispose of the material as indicated by the arrows in Fig. 4, should for his own reasons make the disposition as indicated in Fig. 6, then the haul should be calculated on the assumption that the material was hauled as indicated by the arrows in Fig. 4. This applies whether *xy* is less than the free-haul distance (Secs. 40, 44) or not.

If the fills be made by dumping from trestles we have in this case (Fig. 6) not only the crosshaul above indicated but also the crosshaul pointed out in the examples of the preceding section.

4. **Swell and shrinkage.** — The act of excavating the material of a cut causes the material to expand in volume, because of the separating of the parts. When the material is dumped in the fill there is usually some reduction of the increased volume, due to the consolidation of the parts. See Gillette's "Earthwork and Its Cost," pp. 11–18; Gillette's "Rock Excavation," pp. 8–11;

"Manual" of the Am. Ry. Engrg. Assoc., 1911, p. 35; Webb's "Railroad Construction," pp. 114–118, Arts. 96, 97; Prelini's "Earth and Rock Excavation," pp. 328–331.

Uniform unmixed material, as sand, or clay, or loam, or sandstone, etc., handled in a uniform way, under uniform conditions, may be expected to have a constant relative change of

Method of Handling	Weather	PURE MATERIALS (*a*)=placed in thin horizontal layers (*b*)=dumped from top of fill over side or end																	
		Loam and Vegetable Matter		Loam		Sand		Gravel		Clay		Shale		Sandstone		Limestone		Granite	
		a	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
Shovels	dry																		
	wet																		
Wheelbarrows	dry																		
	wet																		
Scrapers	dry																		
	wet																		
Wheel Scrapers	dry																		
	wet																		
Carts	dry																		
	wet																		
Dump Wagons	dry																		
	wet																		
Cars	dry																		
	wet																		

Fig. 6a.

volume, due to excavation, hauling, and dumping. After the material is placed in the fill its volume decreases, as a rule, for some months or years. Fills of uniform unmixed material, made under uniform conditions, in a uniform way, may be expected, with continuing uniform conditions, to contract with age according to the same law. The foregoing with regard to uniform unmixed materials applies as well to uniform mixtures of different materials. By observation it is possible to make out a schedule like that above for material which is placed in fills of like dimensions. The accompanying schedule, it will be observed, is for fills of one height only. For fills of another height it would be necessary to make a corresponding similar schedule. Perhaps five such schedules would be sufficient for heights of fill running up to 100 feet. These five schedules would give data for cases of unmixed materials only. The foregoing, taken in connection with the following section and with the further fact that mixtures of endless variety are met in practice, will account for the hesitancy of writers to give hard-and-fast rules for predicting swell.

5. Swell of two or more materials depends on thoroughness of mixing. — Fig. 7 shows a layer of 1000 c.y. of unmixed earth

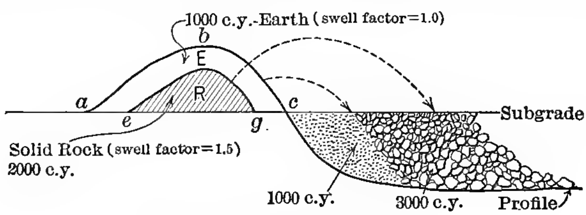


Fig. 7.

lying over 2000 c.y. of unmixed solid rock. Fig. 8 shows the same cut. The earth is supposed to occupy the same space in

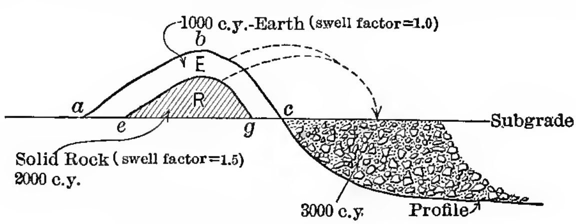
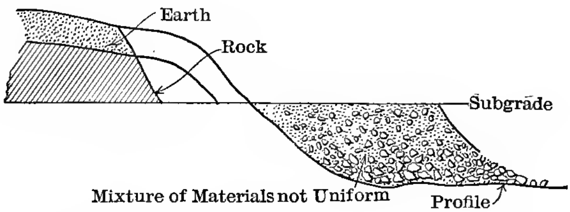


Fig. 8.

the fill as in the cut, that is, the earth is supposed neither to swell nor shrink. The solid rock is supposed to swell so that 1 c.y. of cut makes $1\frac{1}{2}$ c.y. of fill.

Fig. 7 shows the result of keeping the rock and earth separate when making the excavation and fill. The total fill measures 4000 c.y. The total cut is 3000 c.y. Hence in Fig. 7 the increase in the volume of the material of the cut when deposited

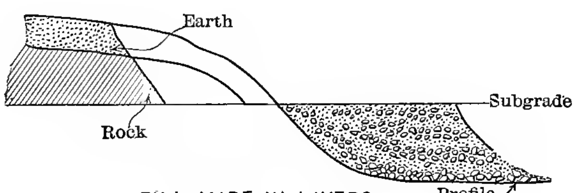


FILL MADE BY CARTS DUMPING OVER THE END

FIG. 9.

in the fill is 1000 c.y.; that is, the swell ratio of the cut taken as a whole is $\frac{1}{3}$ (Sec. 6).

Fig. 8 shows the effect of placing in the fill the material of the cut, uniformly mixed. The 1000 c.y. of voids in the mass of broken rock is precisely filled with the 1000 c.y. of earth.



FILL MADE IN LAYERS

FIG. 10.

Hence in Fig. 8 the fill measures 3000 c.y. Hence the material from the fill occupies the same total space as it occupied in the cut. In Fig. 8, then, we have neither swell nor shrinkage in the grading as a whole.

It may be objected that Figs. 7 and 8 show extreme cases which are not to be expected in practice. They do show the limiting cases: in Fig. 7 we have no mixture; in Fig. 8 a complete mixture. Cases in practice, then, will be of some degree of mixture, or, rather, of various degrees of mixture, approaching the one limit or the other, depending on the plant and methods used (see Figs. 9, 10, 11). The point made here is that it is not usually possible to predict with close approximation the swell of a cut composed of two or more materials, even if we know the swell ratio of each of the component materials.

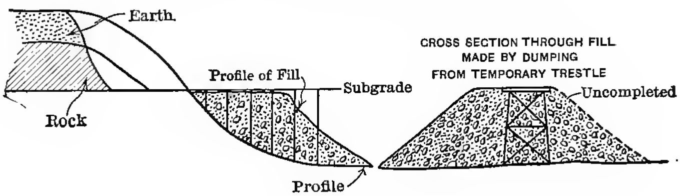


FIG. 11.

6. Swell increment; swell ratio; swell factor. — The "swell increment" of a body of material is the increase in bulk resulting from shifting the material from its original position to a fill. Swell increment is usually expressed in cubic yards. If there is shrinkage, the increment is negative.

Let C = volume (c.y.) of a given body of material in place, (that is, in the cut before being disturbed).

F = volume (c.y.) of the same body of material after deposit in fill;

i = swell increment;

r = swell ratio $= \dfrac{\text{swell increment}}{\text{volume in place}}$;

s = swell factor.

Then the swell increment is

$$i = F - C. \tag{2}$$

The "swell ratio" is

$$r = \frac{i}{C}. \tag{3}$$

The "swell factor" is

$$s = \frac{F}{C} = 1 + r. \tag{4}$$

The swell factor is the reciprocal of the equating factor (Sec. 7). (For practical determination of swell factor, see Secs. 8, 9.)

Example 1. — A fill which measures 5000 c.y. is made from a cut which measures 4000 c.y. The swell increment is $i = F - C = 5000 - 4000 = 1000$. The swell ratio is $r = \frac{i}{C} = \frac{1000}{4000} = 0.25$, or 25%. The swell factor is $s = \frac{F}{C} = \frac{5000}{4000} = 1.25$.

Example 2. — A fill measuring 350 c.y. is made from a cut measuring 400 c.y. In this case the swell increment is $i = F - C = 350 - 400 = - 50$ c.y (This negative increment, of course, means shrinkage.) The swell ratio is $r = \frac{i}{C} = -\frac{50}{400} = - 0.125$, or $- 12.5\%$. (This negative ratio would commonly be referred to as twelve and a half per cent shrinkage.) The swell factor is $s = \frac{F}{C} = \frac{350}{400} = \frac{7}{8} = 0.875$.

7. **Equating factor.** — The equating factor is the reciprocal of the swell factor.

Let C = volume (c.y.) of a given body of material in place.
F = volume (c.y.) of the same body of material in fill.
q = equating factor.

Then the equating factor is

$$q = \frac{C}{F} = \frac{1}{s}. \tag{5}$$

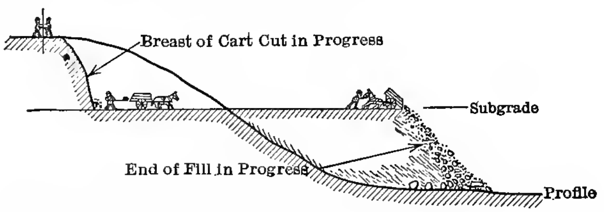
The equating factor is that by which a given volume of fill must be multiplied to obtain the volume of the same body of material in place.

Example. — A definite portion of a fill measures 500 c.y., and the definite portion of the cut originally occupied by that 500 c.y. measures 400 c.y. What is the equating factor?

The equating factor is $q = \frac{C}{F} = \frac{400}{500} = 0.8$. The swell factor is $s = \frac{1}{0.8} = 1.25$.

8. **Determination of swell factor; simple case.** — Fig. 12 shows a simple cart-cut in progress. Both the cut and the fill

are made to full section as the work proceeds. At any time during the progress of this work it is easy to cross-section the cut and the fill as far as completed at that time, and thus obtain the volume of the cut and of the fill, and from these the swell factor. The cut is cross-sectioned at the end of the month for the monthly estimate, and if the extra work of cross-sectioning the fill is done at that time, the most accurate data for computing the swell factor are had at minimum expense.



VOLUME READILY MEASURED IN PLACE AND IN FILL

FIG. 12.

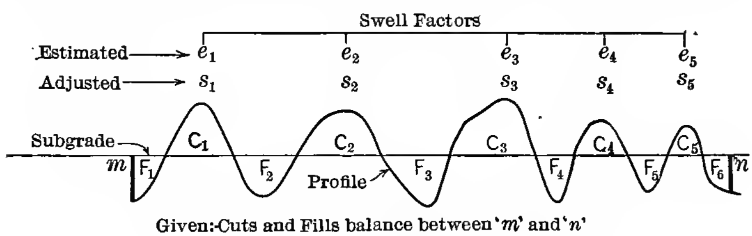
In many cases, however, the fill is not carried forward at full height and width, but in such manner that the work of cross-sectioning would take considerable time — more time often than could be given without providing an extra force for the surveying party.

Again, a fill may be composed of materials brought from two or more cuts and deposited in such manner as to intermingle the materials from the different sources. For example, the output of two steam-shovels, working in two cuts, say, one of clay and the other of solid rock, may be used to widen the same fill. In such a case it is impracticable to measure the volume of that portion of the fill which comes from either cut.

Finally, of two trains serving a steam-shovel, one may be dumping on one fill and the other on another fill. If the material going to each fill be deposited in well-defined position so as to be readily cross-sectioned we can, by counting the number of carloads going to each fill, and multiplying by the average carload in cubic yards, obtain approximately the volume of cut hauled to each fill. And then, by measuring the fills, we can compute an approximate swell-factor. If, however, in this case, either train add an irregular layer to a ragged fill previously made in part from another cut, it will be impracticable to obtain cross-sections during the progress of the work which will give satisfactory swell factors.

The method of determining the swell factor in these complex cases will be given in the next section.

9. Determination of swell factor; complex case. — When the work of grading a stretch of railroad is so carried on that it is



ADJUSTMENT OF ESTIMATED SWELL FACTORS

FIG. 13.

impracticable to take such cross-sections as will enable one to compute directly the swell factor for any single cut or part of a cut, we have to resort to estimating the swell factor of each cut (or part of cut) and so adjusting the estimated swell factors as to make them harmonize with the fact (ascertained by cross-sectioning all the cuts and fills of the stretch in question after they are completed) that so many (total) cubic yards of cut have made so many (total) cubic yards of fill.

Let us illustrate. Referring to Fig. 13 let us assume that the grading has been completed and that between the points m and n the cuts precisely make the fills; in other words, the cuts and fills balance between m and n. Let it be assumed that during the progress of the work it has been impracticable to make measurements from which to obtain the swell factor of any single cut or any part of a cut. Required to determine the swell factor for each of the cuts of the series.

Having no measurements from which to compute the swell factor of any one of the cuts, we are compelled to estimate the swell factor of each cut, basing the estimate on knowledge gained from past experience and on the knowledge of the material of the cut and of the method and conditions of handling the material.

Let e_1 = estimated swell-factor of cut C_1;
e_2 = estimated swell-factor of cut C_2;
e_3 = estimated swell-factor of cut C_3.
. . . .

Now these estimated swell-factors must be adjusted to satisfy the known condition, viz.: The sum of the products formed by multiplying the volume of each cut by its swell factor must exactly equal the sum of the volumes of the fills.

The most reasonable adjustment of the estimated swell-factors is that which maintains their relative values.

Let s_1 = adjusted swell-factor of cut C_1;
s_2 = adjusted swell-factor of cut C_2;
s_3 = adjusted swell-factor of cut C_3.
. . . .

Let C_1 = volume (c.y.) of cut C_1, as computed from final measurements;
C_2 = volume (c.y.) of cut C_2, as computed from final measurements;
C_3 = volume (c.y.) of cut C_3, as computed from final measurements.
. . . .

Let F_1 = volume (c.y.) of fill F_1, as computed from final measurements;
F_2 = volume (c.y.) of fill F_2, as computed from final measurements;

F_3 = volume (c.y.) of fill F_3, as computed from final measurements.

. . . .

Now if the factors are adjusted so as to maintain the same relative values, we have

$$\left.\begin{array}{l} \frac{s_2}{s_1} = \frac{e_2}{e_1} = k_2, \text{ say;} \\ \frac{s_3}{s_1} = \frac{e_3}{e_1} = k_3, \text{ say;} \\ \frac{s_4}{s_1} = \frac{e_4}{e_1} = k_4, \text{ say;} \\ \ldots \end{array}\right\} \quad (6)$$

whence,

$$\left.\begin{array}{l} s_2 = k_2 s_1, \\ s_3 = k_3 s_1, \\ s_4 = k_4 s_1. \\ \ldots \end{array}\right\} \quad (7)$$

From the known condition that the cuts and fills balance, we have

$$C_1 s_1 + C_2 s_2 + C_3 s_3 + \cdots + C_5 s_5 = F_1 + F_2 + \cdots + F_6, \quad (8)$$

which by substitutions from eq. 7 becomes

$$C_1 s_1 + C_2 k_2 s_1 + C_3 k_3 s_1 + \cdots + C_5 k_5 s_1 = F_1 + F_2 + F_3 + \cdots + F_6. \quad (9)$$

Solving this equation for s_1 we have

$$s_1 = \frac{F_1 + F_2 + F_3 + \cdots + F_6}{C_1 + C_2 k_2 + C_3 k_3 + \cdots + C_5 k_5}. \quad (10)$$

Having the value of s_1, the values of s_2, s_3, s_4, . . . are computed by eq. 7.

The foregoing method of fixing upon the swell factors for the several cuts may be divided for convenient use into the following steps:

Step 1. Estimate the swell factors e_1, e_2, e_3, . . . for the cuts C_1, C_2, C_3, . . . respectively.

Step 2. Compute

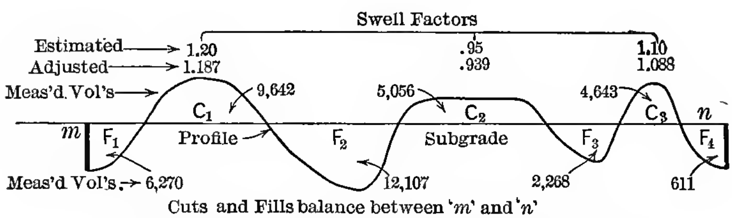
$$\left.\begin{aligned} k_2 &= \frac{e_2}{e_1}, \\ k_3 &= \frac{e_3}{e_1}, \\ k_4 &= \frac{e_4}{e_1}. \\ &\cdots \end{aligned}\right\} \qquad (11)$$

Step 3. Compute the adjusted swell-factor for cut C_1:

$$s_1 = \frac{F_1 + F_2 + \cdots + F_6}{C_1 + C_2k_2 + C_3k_3 + \cdots + C_5k_5}. \qquad (12)$$

Step 4. Compute the adjusted swell-factor for each of the remaining cuts:

$$\left.\begin{aligned} s_2 &= k_2 s_1, \\ s_3 &= k_3 s_1, \\ s_4 &= k_4 s_1. \\ &\cdots \end{aligned}\right\} \qquad (13)$$



EXAMPLE OF ADJUSTMENT OF SWELL FACTORS

FIG. 14.

To illustrate we turn to the following example. Referring to Fig. 14: The grading between m and n has been completed. The final measurements of cuts C_1, C_2, C_3 and fills F_1, F_2, F_3, F_4 have been made; and from these measurements have been computed the volumes shown in Fig. 14.

Step 1. According to the best available judgment the swell ratios have been estimated to be as follows: swell ratio of cut C_1, 0.20; of C_2, -0.05; and of C_3, 0.10. Hence

Estimated swell-factor of cut C_1 is $e_1 = 1.20$.
Estimated swell-factor of cut C_2 is $e_2 = 0.95$.
Estimated swell-factor of cut C_3 is $e_3 = 1.10$.

Step 2. By the use of eq. 11, and a slide-rule. the following multipliers are computed:

$$k_2 = \frac{e_2}{e_1} = \frac{0.95}{1.20} = 0.792,$$

$$k_3 = \frac{e_3}{e_1} = \frac{1.10}{1.20} = 0.917.$$

Step 3. The adjusted swell-factor for cut C_1 is (eq. 12)

$$s_1 = \frac{F_1 + F_2 + F_3 + F_4}{C_1 + C_2k_2 + C_3k_3}$$
$$= \frac{6270 + 12{,}107 + 2268 + 611}{9642 + 5056\,(0.792) + 4643\,(0.917)}$$
$$= 1.187.$$

Step 4. The adjusted swell-factors for the remaining cuts are computed by eq. 13.

$$s_2 = k_2s_1 = 0.792 \times 1.187 = 0.939.$$
$$s_3 = k_3s_1 = 0.917 \times 1.187 = 1.088.$$

To check the foregoing computations this test is applied:

The sum of the products formed by multiplying each cut-volume by its swell-factor must equal the sum of the corresponding fill-volumes, that is,

$$C_1s_1 + C_2s_2 + C_3s_3 = F_1 + F_2 + F_3 + F_4.$$

$$C_1s_1 = 9642 \times 1.187 = 11{,}440$$
$$C_2s_2 = 5056 \times 0.939 = 4{,}750$$
$$C_3s_3 = 4643 \times 1.088 = 5{,}050$$

$$C_1s_1 + C_2s_2 + C_3s_3 = 21{,}240$$
$$F_1 + F_2 + F_3 + F_4 = 21{,}256$$

Discrepancy 16

Thus the test is met; the work checks; the discrepancy of 16 c.y., which is due to using the slide-rule, is negligible.

The uncertainty in the estimated swell-factors is usually such as to permit the use of the slide-rule in making all the computations of this section.

CHAPTER II

THE MASS CURVE

This chapter sets forth the principles of construction and interpretation of the mass curve; describes the horizontal and the oblique balancing line; and discusses the effect of drafting-errors on results obtained from the mass curve.

10. **Construction of mass curve or profile of quantities.**—The steps taken to construct the mass curve for the profile *ab* shown in Fig. 15, are as follows:

Step 1. Data. — The station- and substation-volumes are marked on the profile. Let it be supposed that the cuts have a swell ratio of 0.25. Let it be supposed that the quantities shown on the profile are quantities computed from final measurements of completed grading.

Step 2. Tabulate the stations and substations, and the volumes of cut and fill. See the accompanying tabular form.

Step 3. Multiply each fill-volume by the equating factor $\frac{1}{1.25} = 0.8$ (Sec. 6).

Step 4. Set down in the fifth column, opposite each station and substation, the algebraic sum of cut-volumes and equated fill-volumes between station 12 and that station. For example, the ordinate for sta. 13 is 100; the ordinate for sta. 14 is $100 + 200 = 300$; for sta. 15, is $100 + 200 + 50 - 80 = 270$; and so on.

Step 5. Now taking some heavy horizontal line *AB* of the profile paper as a base line for mass-curve ordinates, lay off to some scale (1 inch = 200 c.y., here) at each station the computed ordinate for that station. Thus at sta. 13 lay off above the base line *AB* 100 c.y. (that is, one-half inch); at sta. 14 lay off above the base line 300 c.y.; and so on. Join the upper ends of adjacent ordinates thus laid off, by straight lines. The broken line thus formed is the mass curve or profile of quantities.

NOTE. — In the illustration above the station-volumes have been summed from left to right and thus plotted. It would be just as well to sum and plot from right to left. Also, we might call fills plus and cuts minus, and the resulting mass curve would serve all purposes just as well as with cuts called plus and fills minus.

COMPUTATION FOR ORDINATES OF MASS CURVE

Station.	Vol. (c.y.)		Equated fill = fill × (.8) (−).	Ordinates (c.y.).
	Cut (+).	Fill.		
12				
	100			
13				100
	200			
14				300
	50			
14+40				350
		100	80	
15				270
		375	300	
16				− 30
		200	160	
16+70				−190
	20			
17				−170
	140			
18				− 30
	20			
18+20				− 10
		120	100	
19				−110

From the foregoing we make the following definition: a mass curve is a curve of which the abscissas are stations, and the ordinate at any station (or substation) is the algebraic sum of the cut- and fill-volumes (calling cut plus and fill minus, or vice versa) between that station (or substation) and some chosen initial point of the profile. See Fig. 15.

Before computing the ordinates of the mass curve either (1)

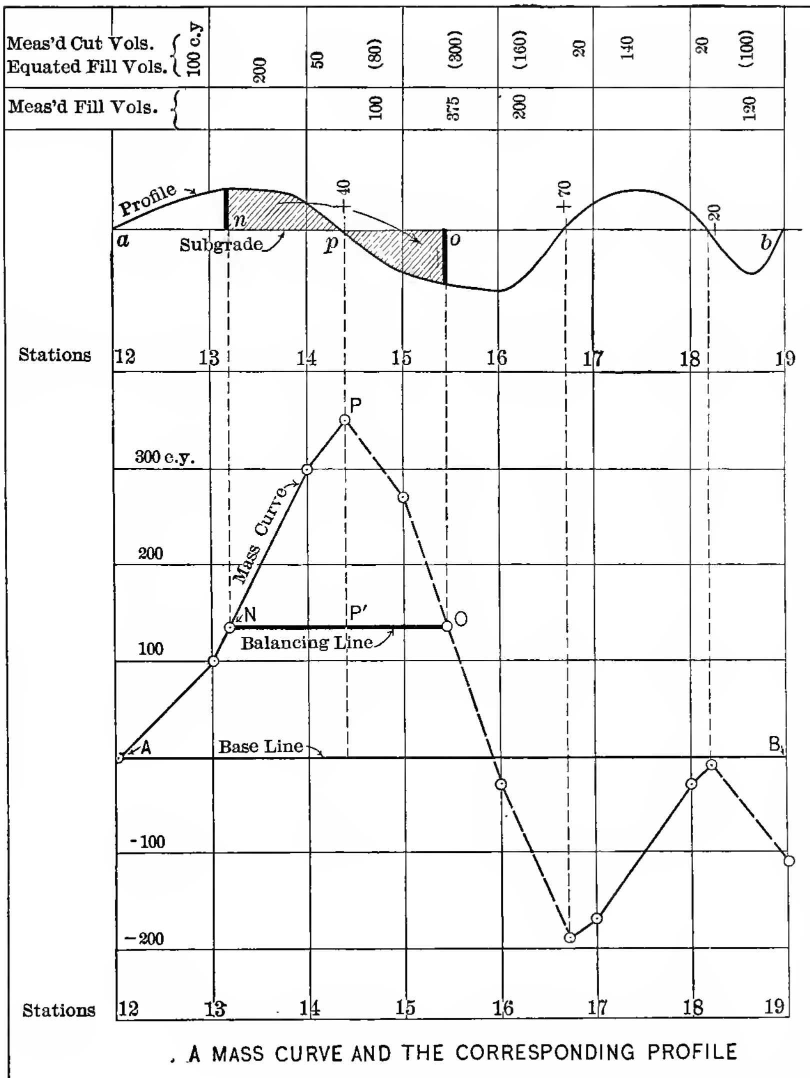

Meas'd Cut Vols.
Equated Fill Vols.
100 c.y
200
50
(80)
(300)
(160)
20
140
20
(100)
Meas'd Fill Vols.
100
375
200
120
Profile
n
a
Subgrade
p
o
+40
+70
-20
b
Stations
12
13
14
15
16
17
18
19
P
300 c.y.
Mass Curve
200
N
P'
O
Balancing Line
100
A
Base Line
B
-100
-200
Stations
12
13
14
15
16
17
18
19
A MASS CURVE AND THE CORRESPONDING PROFILE

Fig. 15.

the cut-volumes for the several station-intervals must be multiplied by their proper swell-factors (Sec. 6); or (2) the fill-volumes for the several station-intervals must be equated to volume in place (Sec. 7). (See exception to this statement in Sec. 12.)

If the grading has been completed and the volume computed from final measurements, then the fill-volumes should be equated to volume in place, and the equated fill-volumes used with the cut-volumes in computing ordinates of the mass curve; but if the grading has not been completed, or has not been started, the cut-volumes must be multiplied by the proper swell-factors before the ordinates of the mass curve are computed, and the swelled cut-volumes used with the fill-volumes to obtain the ordinates. When there are both cut and fill in any station-interval — as in side-hill work — the excess of the one over the other is used as an increment (positive or negative as the case may be) in computing mass-curve ordinates.

11. **Characteristics of the mass curve.** — A study of Fig. 15 will show the truth of the following statements:

Within the limits of a single cut the mass curve rises from left to right. Within the limits of a single fill the mass curve falls from left to right. Hence in passing from left to right from cut to fill we have at the grade point a maximum ordinate for the mass curve; and in passing from left to right from fill to cut we have at the grade point a minimum ordinate.

NOTE. — Had the volumes been summed from right to left in the computation for ordinates, the resulting mass curve would rise from right to left instead of from left to right within the range of a single cut. If cuts had been reckoned minus and fills plus, while retaining the left-to-right summation, the mass curve would slope downward from left to right within the range of a single cut.

The algebraic difference between the ordinates of any two stations which lie between a maximum point of the mass curve and an adjacent minimum point, represents the yardage between the two stations.

The slope of the mass curve is steepest at stations of greatest volumes. See Fig. 38 where both mass curve and profile are drawn to scale.

Other characteristics of the mass curve are given in Secs. 12, 13, 14, 15.

12. The horizontal balancing line. — The mass curve in Fig. 15 is plotted from cut-volumes and equated fill-volumes. Any horizontal line drawn to cut off a loop of such a mass curve cuts the curve in two points between which the cut will precisely make the fill. Thus the horizontal line *NO* cuts the curve at *N* and *O*. Between *N* and *O* the cut just makes the fill. Such a line is called a "balancing line." Under the given conditions of plotting the mass curve of Fig. 15, a loop above the balancing line indicates forward hauling, and a loop below the balancing line indicates backward hauling. See Figs. 50–60.

It is evident that all balancing lines are horizontal for a mass curve which is plotted from cut-volumes and equated fill-volumes or from fill-volumes and equated (i.e., swelled) cut-volumes.

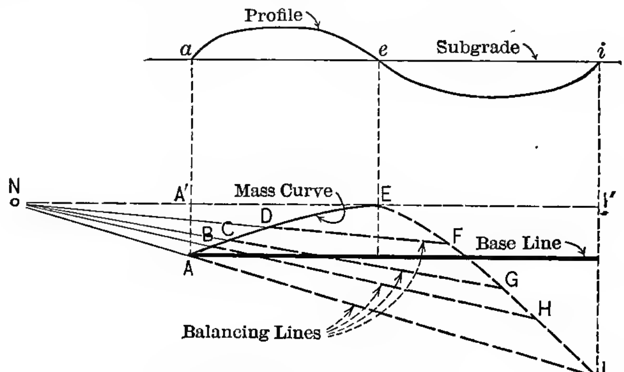
When the mass curve is plotted from cut-volumes and actual fill-volumes a balancing line will be oblique if the material from the cut either swells or shrinks. See Sec. 13.

The horizontal balancing line only is used in the problems of this book.

13. The oblique balancing line. — Fig. 16 shows the profile of a cut and fill. Let it be assumed that the material of the cut swells, and that the cut just makes the fill. The mass curve below the profile is plotted from the cut-volumes and actual fill-volumes — the volumes in fill have not been equated to volumes in place. Thus the total fill-volume *I′I* is greater than the total cut-volume *AA′*. Where the mass curve is thus plotted without first equating the fill-volumes to volumes in place, or equating the cut-volumes to fill-volumes, the balancing line will be a horizontal line only when the equating-factor is unity — only when the material of the cut neither shrinks nor swells.

The balancing lines in Fig. 16 are oblique. The balancing line which passes through A passes also through I, because between a and i the cut just makes the fill.

Produce the line AI to meet at N the horizontal line EN drawn through the highest point E of the mass curve. Each of the lines BH, CG, DF radiating from N cuts the mass curve in two points and is approximately a balancing line.



OBLIQUE BALANCING LINE

FIG. 16.

It is the practice of some engineers to plot the mass curve from the unequated cut- and fill-volumes and to use, therefore, the oblique balancing line. An example of the use of the oblique balancing line is given in an article, by Mr. S. B. Fisher, printed in the *Engineering News*, Jan. 31, 1891, under the title "Estimating Overhaul in Earthwork by Means of the Profile of Quantities." Mr. Fisher further explained the oblique balancing line in a letter printed in the *Engineering News*, Feb. 7, 1891. Both article and letter are reprinted in Gillette's "Earthwork and Its Cost," pp. 217–225; and in Proceedings, American Railway Engineering Association, vol. 7, 1906, pp. 381–385.

14. **Relation between haul and mass-curve area when the mass curve is plotted from cut-volumes and equated fill-volumes.** — The mass curve of Fig. 15 is plotted from cut-volumes and

equated fill-volumes. The area lying between the balancing line and the corresponding loop of the mass curve is the measure of the haul (Sec. 1) involved in making the cut and fill between the extremities of the loop. Thus the balancing line NO cuts the mass curve at N and O, and cuts off the loop NPO. The area of the loop NPO is a measure of the haul performed in making the fill between p and o from the cut between n and p. The extreme haul-distance for the material between n and o is $no = NO$; and the volume of cut np is equal to PP' (to scale) — equals equated fill-volume between p and o.

To convert the area NPO into haul (sta-yds.) multiply the area NPO (in square inches) by the product formed by multiplying the stations represented by one horizontal inch of paper by the number of cubic yards represented by one vertical inch of paper.

Let Y = number of cubic yards represented by one inch of ordinate;
S = number of feet represented by one inch of abscissa;
A = haul area in square inches;
H_1 = haul in station-yards represented by one square inch of area.

Then

$$H_1 = \frac{S}{100} Y. \tag{14}$$

The area, A, of the loop NPO measured with the planimeter, is 2.8 sq. in. For Fig. 15, Y = 200 c.y., and S = 200 ft. Hence the haul is

$$H = AH_1 = 2.8 \times \tfrac{200}{100} \times 200 = 1120 \text{ sta-yds.}$$

15. Relation between haul and mass-curve area when mass curve is plotted from fill-volumes and equated cut-volumes. — In Fig. 17 the cut is just sufficient to make the fill. The volume of the cut is C c.y., and the swell-factor is s. The volume of the fill is F c.y. Then $C = \frac{F}{s} = Fq$, and $Cs = F$, where q is the equating-factor of the fill (Sec. 7).

First, we will reduce the station-volumes of fill to volumes in place by multiplying each station-volume by q, and construct the mass curve above the profile, using the cut-volumes and equated

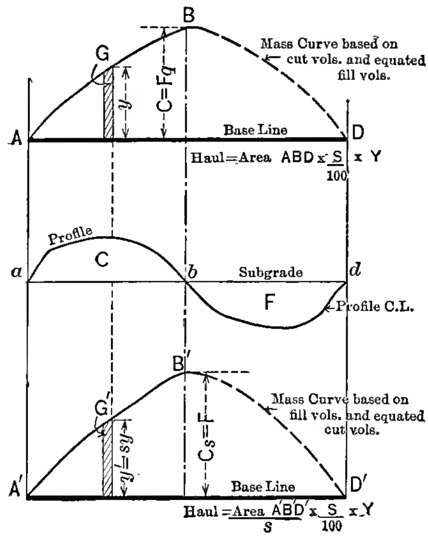


FIG. 17.

fill-volumes. Now the maximum ordinate of the mass curve ABD is C (to scale); and the area of the mass curve ABD represents the haul. The haul involved in the grading is

$$H = (\text{area } ABD \text{ in sq. in.}) \frac{S}{100} Y \text{ sta-yds.} \qquad (14a)$$

Next, let us equate the station-volumes of cut to fill-volumes by multiplying each station-volume of cut by the swell-factor s; then plot the mass curve $A'B'D'$ below the profile, using the fill-volumes and equated cut-volumes. In this mass curve the maximum ordinate is equal (to scale) to $Cs = F$. Now for every vertical strip, as G, of the area of the upper mass curve there is a corresponding strip G' of the lower mass curve. The two strips being of the same width their areas are proportional to their heights. If the height of G is y the height of G' is $y' = sy$. Hence the area of G' is s times the area of G; and hence the area $A'B'D'$ of the lower mass curve is s times the area of the upper mass curve. And, since the total haul expressed in terms of the upper mass curve is

$$H = (\text{area } ABD \text{ in sq. in.}) \frac{S}{100} Y,$$

the expression for the total haul in terms of the lower mass curve is

$$H = \frac{(\text{area } A'B'D' \text{ in sq. in.})}{s} \frac{S}{100} Y. \tag{14b}$$

Since an estimate of haul based on a mass curve is subject to errors resulting from the errors of plotting and scaling, it is well to know the relation which exists between errors of drafting and the resulting errors in areas. This matter is taken up in the two sections following.

16. Limit of error in a distance plotted or scaled. — We distinguish between an error and a mistake. A mistake results from incorrect work. An error results from lack of precision in the work. If we lay off 2 in. when we should lay off $2\frac{1}{4}$ in., we make a mistake or blunder. When we try to lay off $AB = 2$ in. from a given point along a given line, we try to place the zero scratch of the scale precisely at the mark A; but we know that, owing to the width of the scratch and of the mark A and the limitations of vision, the center of the scratch may be as much as, say, $\frac{1}{200}$ in. one side or the other of the center of the mark A; that is, we know that the position of the zero scratch may be in error by as much as $\frac{1}{200}$ in. Likewise, when we try to make the mark B so that its center shall be precisely opposite the 2-in. scratch of the scale, we know that the resulting mark, B, may have its center as much as $\frac{1}{200}$ in. to one side of the center of the 2-in. scratch. In other words, we know that there may be, due to lack of precision in placing the mark *B*, an error of as much as $\frac{1}{200}$ in. in the position of *B*. Thus the resulting error in the plotted distance *AB* may be as much as $\frac{2}{200}$ in. = 0.01 in.

If the two errors are each $\frac{1}{200}$ in. and both outward, then the distance between mark *A* and mark *B* is 2.01 in. If the two errors are each $\frac{1}{200}$ in. and both inward, the distance between *A* and *B* is 2.00 − 0.01 = 1.99 in. And if the two errors are equal but one is inward while the other is outward, the two errors cancel and the distance from mark *A* to mark *B* is just 2 in. In general, if in placing the zero scratch of the scale against a mark and if in placing a mark on the paper against a given scratch on the scale we are subject to an error of *e* in., then the limit of error in any distance which we lay off at one application of the scale is $2e$.

The statements above apply as well to the work of scaling a distance from a drawing. Therefore when a distance is scaled from a drawing the scaled distance is subject to an error which is the resultant of the errors of plotting and of scaling. Hence the limit of error *E* in a scaled distance, due to the limit of error in placing a scratch of the scale against a mark, is

$$E = 4e \text{ in.} \tag{15}$$

If one inch of paper represents *S* feet,

$$E = 4eS \text{ ft.} \tag{16}$$

Example. — A draftsman lays off on paper a distance *AB* on the scale of 1 in. = 400 ft. We scale the distance *AB* and find it to be 875 ft. What is the limit of error in the 875 ft. if in plotting and in scaling there is a possible error of 0.01 in. in placing a scratch of the scale against a mark on the paper? The answer is

$$E = 4eS = 4\,(0.01)\,400 = 16 \text{ ft.}$$

17. Limit of error in area due to errors in distance. — Suppose the true dimensions of the rectangle *ABCD* (Fig. 18) are *d* inches by a inches. Suppose that in scaling the dimensions from the drawing we make an error E in each, so that we come to have $d + E$ and $a + E$ as the dimensions of the rectangle. The true area of the rectangle is da. The area obtained by use of the scaled dimensions is $(d + E)(a + E) = da + dE + aE + E^2$. Ignoring E^2, which is relatively small $(da + dE + aE) - da, = (dE + aE)$, is the limit of error in area, due to the error E in each dimension. If E' is the limit of error in area, due to E, the limit of error in distance, we have

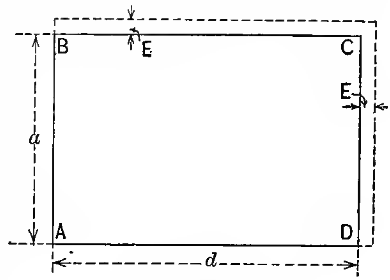


FIG. 18.

$$E' = E(d + a) \text{ sq. in.,} \tag{17}$$

where E, d, and a are in inches.

If each inch of paper represents S feet, then each square inch of paper represents S^2 square feet,

and
$$E' = E(d + a) S^2 \text{ sq. ft.} \tag{18}$$

Substituting for E its value (eq. 15) in terms of e (the limit of error in placing a scratch of the scale against a mark on the paper), eq. 18 becomes

$$E' = 4e(d + a) S^2 \text{ sq. ft.} \tag{19}$$

If the vertical scale of the rectangle is 1 in. = Y c.y., and the horizontal scale is 1 in. = S ft., then 1 sq. in. of paper represents $Y \dfrac{S}{100}$ sta-yds. of haul,

and
$$E' = 4e(d + a) Y \frac{S}{100} \text{ sta-yds.} \tag{20}$$

Example. — A mass curve is drawn with vertical scale of 1 in. = 1000 c.y., and horizontal scale of 1 in. = 400 ft. A rectangular area representing haul is scaled. The horizontal dimension appears to be 6.4 in.; and the vertical, 1.5 in. The resulting haul is $H = A \frac{S}{100} Y$ (eq. 14a) $= 6.4 \times 1.5 \times \frac{400}{100} \times 1000 = 38{,}400$ sta-yds. If in plotting and scaling the mass curve the limit of error in placing a scratch of the scale against a mark on the paper was 0.01 in., what is the resulting limit of error in the 38,400 sta-yds.? The answer is $E' = 4\,e\,(d + a)\,Y \frac{S}{100} = 4\,(0.01)\,(6.4 + 1.5)\,1000 \frac{400}{100} = 1264$ sta-yds.

We have been considering the limiting error. It should be borne in mind that limiting errors in drafting are infrequent, and that the errors are as likely to be positive as negative, and therefore tend to neutralize one another.

18. Scale of ordinates for mass curve. — We may say that anything less than $\frac{1}{200}$-inch distance on a profile is inappreciable. Hence, for a scale of 1 in. = 100 c.y. less than 0.5 c.y. is negligible; for 1 in. = 1000 c.y. less than 5 c.y. is negligible; for 1 in. = 5000 c.y. less than 25 c.y. is negligible; and so on. The larger the scale the less the uncertainty, due to the drawing itself, in the results obtained through the use of the mass curve. On the other hand, the larger the scale the greater the required width (top to bottom) of the paper on which the mass curve is drawn. The choice of scale for the ordinates of the mass curve in any given case will depend (1) on the arithmetic sum of the maximum positive ordinate and maximum negative ordinate of the curve; (2) on the horizontal scale used; (3) on the uncertainty of the data upon which the ordinates are computed; (4) on the desired accuracy of the results to be obtained from the use of the mass curve; and (5) on the convenient maximum top-to-bottom dimension of the paper.

When drawing a mass curve for cuts and fills of great volume, the scale used for the ordinates is usually a compromise. In any case the scale should be no larger than necessary to keep the errors from the use of the mass curve well within the limits of error in the data upon which the computed ordinates are based. With great volumes the limit of error in the data is comparatively great owing to the irregular manner of excavating and depositing the material (Secs. 2, 3, 4).

19. Plotting the mass curve. — The mass curve is most conveniently plotted and used when plotted on profile paper of the same horizontal scale as the profile of the line under consideration.

For "Plate A" profile paper (20 horizontal rulings to the inch), the mass curve ordinates, — provided the scale used therefor is 200 or 2000 or 20,000 c.y. to the inch, — are most readily plotted in the manner of plotting points on the profile. For "Plate B" profile paper (30 horizontal lines to the inch), and for a scale of 300, or 3000 or 30,000 c.y. to the inch for ordinates, the mass-curve points are most quickly plotted in the manner of plotting profile points. Under other conditions the mass-curve ordinates are most conveniently laid off by means of the engineers' scale, ignoring the horizontal rulings of the profile paper. When the engineers' scale is to be used instead of the rulings of the profile paper in laying off the ordinates of the mass curve, the scale of ordinates should, for convenience, be one of the following: 100, 1000, 10,000 or 100,000 c.y. to the inch; 200, 2000 or 20,000 c.y. to the inch; 300, 3000, or 30,000 to the inch; 400, 4000, or 40,000 to the inch; 500, 5000, or 50,000 to the inch; or 600, 6000, or 60,000 c.y. to the inch.

When plotting the mass curve and drawing lines by means of which to obtain results from it, keep the pencil as sharp as a needle. The 6-H pencil is best if the eyes of the plotter and the light are good; otherwise use a 4-H pencil although this will require frequent sharpening, and the lines will rub somewhat. Sharpen the pencil to a long cone point on emery paper, and polish the point on a piece of detail paper. Repolish the point after drawing each foot or two of line; and the knife and emery paper will have to be used only occasionally. The importance of drawing the finest lines and making the finest points in all graphical computation must be fully appreciated by the draftsman;

otherwise second-class work will result. See Secs. 16 and 17. For the sake of convenience use ink of one color for ascending segments of a mass curve, and ink of another color for descending segments; or, use a full line for the one, and a broken line for the other, as shown in Fig. 15.

CHAPTER III

LIMITS AND CENTER OF MASS OF A BODY OF MATERIAL

This chapter describes how, by eye, by arithmetic, and by mass curve, to determine the limits of a body of material in cut or in fill from given conditions; and how to determine the center of volume or mass of such body. (Figs. 19, 21, 21a, and 24 face p. 96.)

20. Limit of a fill made from a given body of material. — Fig. 19. Let it be assumed that the material between sta. 9 and the grade point (sta. 13 + 75) is to be used to make a portion of the adjacent fill. Further let it be assumed that the material swells 25% (swell ratio = 0.25), and that the station-volumes are as entered on the lower part of Fig. 19. The problem is to find the point on the profile to which the material between 9 and 13 + 75 will make the fill.

(a) *Arithmetical solution.* — The total yardage between stas. 9 and 13 + 75 is 1280. Tabulate the stations of fill and the corresponding station-volumes. See the tabulation below. In the third column enter the equated fill-volumes. (The swell ratio being 0.25, the swell-factor is 1.25, and the equating-factor is, therefore, $\frac{1}{1.25} = 0.8$ (see Secs. 6, 7)). Next, fill out the fourth column by entering opposite each station the sum of equated fill-volumes between that station and the grade point, sta. 13 + 75.

Now looking at the summation column, we see that the 1280 c.y. of material will fill to a point between stas. 18 and 19. The fill to sta. 18 requires 1140 c.y., leaving a surplus of 1280 − 1140 = 140 c.y. to fill beyond sta. 18.

Assuming that the cross-sectional area of the fill is uniform between 18 and 19, the (equated) volume of the fill is $\frac{560}{100}$ = 5.6 c.y. per running foot. At this rate it will require $\frac{140}{5.6} = 25$

running feet of fill beyond sta. 18 to use up the 140 c.y. of surplus material. Thus 18 + 25 is the limit of the fill which the given cut will make. (We have assumed that the end of the fill is bounded by a plane of cross-section, i.e., that the fill is made of full section as far as it goes.)

Station.	Measured fill-vol. (c.y.).	Equated fill-vol. (c.y.).	Summation (c.y.)
13+75			
	25	20	
14			20
	150	120	
15			140
	375	300	
16			440
	375	300	
17			740
	500	400	
18			1140
	700	560	
19			1700

(b) *Mass-curve solution.* — The lower portion of Fig. 19 shows the mass curve *ACDB* corresponding to the profile. The base line of the mass curve is *AB*, and *A* is the origin at sta. 9. The ordinate at sta. 10 is 500 c.y.; at sta. 11, 500 + 300 = 800 c.y.; at sta. 12, 800 + 200 = 1000 c.y.; at sta. 13, 1000 + 200 = 1200 c.y.; at sta. 13 + 75, 1200 + 80 = 1280 c.y.; at sta. 14, 1280 − 20 = 1260 c.y.; at sta. 15, 1260 − 120 = 1140 c.y.; and so on. The ordinate at sta. 19 is − 420 c.y.

To find the limit of the fill made by the cut, 9 + 00 to 13 + 75. Through the point of the mass curve at sta. 9 draw a horizontal to intersect the right half of the mass curve. The point of intersection is the limit of the fill. Thus through the point *A* draw the horizontal *AB* (which is the base line in this case) intersecting the mass curve at *B*. *B* marks the right-hand limit of the fill, and is the point sought. By scaling the plus, we find that *B* lies at sta. 18 + 24.

The mass-curve solution is the same as the arithmetical except in form. The mass-curve solution gives a good check on the arithmetical. Of course the results obtained by the two methods may not agree (they differ by 1 ft. in this case) owing to errors incident to graphical computation. Substantial agreement is sufficient.

To find out how far into a cut we shall have to go to obtain enough material to make a given fill is a problem like the foregoing and is solved in the same way.

(c) *Solution by inspection.* — Lay a pencil on Fig. 19 in a position parallel to the vertical rulings. Shift the pencil slowly toward the right (or left) until it appears that the fill lying between the grade point and the pencil just balances the cut between sta. 9 and the grade point. The station and plus of the pencil at this time is approximately the right-hand limit of the fill. A strip of stiff paper will serve better than a pencil for this operation. Bear in mind that volumes are not proportional to profile areas; volume of cut, or fill, increases faster than center height; cuts and fills do not usually have the same roadbed width or the same side-slopes; and, owing to swell of material, a yard of cut may make more than a yard of fill.

The person who applies this method should be ignorant of the computed limit of the fill; otherwise the result which he obtains by inspection will be biased and of little or no value.

21. Center of mass of single prismoid. — By 'single prismoid' is meant the body of material lying between two adjacent stations (each of which may be a full station or a plus station) in cut or in fill. The center of mass of a single prismoid (as that lying between stations *m* and *n*, Fig. 20) lies in that cross-sectional plane which cuts the prismoid *mn* into two equal volumes.

If the cross-sectional area is uniform from *m* to *n*, then the center of mass of the prismoid *mn* lies midway between stas. *m* and *n*.

If the cross-sectional area decreases as we pass from *m* to *n*, evidently the center of mass lies between the mid-section of *mn* and sta. *m*.

For the work of computing haul and overhaul it is customary to assume that the center of mass of a single prismoid (i.e., of a station-volume or substation-volume) lies midway between the end sections of the prismoid.

22. Center of mass of a series of prismoids. Arithmetical solution. — Let *m*, *n*, *o*, *p*, *q* (Fig. 20) be consecutive stations (or stations and substations) along the profile.

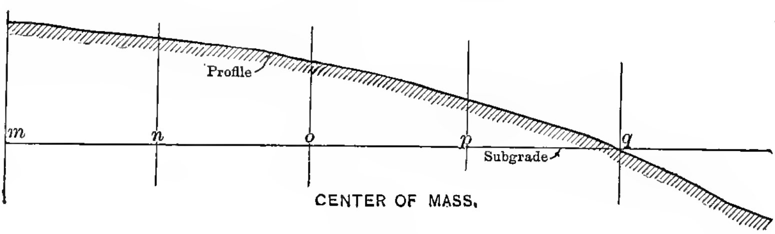


FIG. 20.

If all the prismoids are of uniform cross-sectional area then the center of mass of the series of prismoids, *mn*, *no*, *op*, *pq*, lies midway between stas. *m* and *q*. There are, of course, other exceptional conditions under which the center of mass will lie midway between the extreme stations of the series.

Sometimes the center of mass is assumed to be midway between the extreme stations even in those cases (most frequent) in which the center of mass is known to be actually eccentric (that is, to lie on one side or the other of the mid-point).

The general method of finding the center of mass of a series of prismoids is made clear in the following example:

Example. — Fig. 21 shows station-volumes between stas. 9 and 12 + 28. To find the center of mass of this series of prismoids we proceed as follows: Find the sum of the volumes of the prismoids = 1056 c.y. The half sum is 528 c.y. Then find the sum of the volumes from sta. 9 up to each station. (This work is shown in tabular form.)

Station.	Volume.	Summation.
9		
	500	
10	....	500
	300	
11	....	800
	200	
12	....	1000
	56	
12+28	....	1056
Total	1056 c.y.	
½ total	528	

We find that between sta. 9 and sta. 10 the summation volume is less than one-half the total volume (528) of the series of prismoids; and that between sta. 9 and sta. 11 the summation volume is greater than one-half the total volume of the series. Plainly, then, the center of mass of the series of prismoids lies somewhere between stas. 10 and 11. The center of mass of the series lies 28 c.y. (so to speak) to the right of sta. 10. Now in passing from 10 to 11 we pass 300 c.y., which (assuming that the prismoid 10–11 is of uniform cross-sectional area) is at the rate of $\frac{300}{100} = 3$ c.y. per running foot. To pass over the 28 c.y., then, we must go $\frac{28}{3} = 9.3$ ft. from sta. 10 towards sta. 11. Thus we find the center of mass of the series of prismoids is at sta. 10 + 09.3.

Evidently the result will be the same if we carry the computation from the other end: sta. 12 + 28; thus

Station.	Volume.	Summations.
12+28.		
	56	
12	....	56
	200	
11	....	256
	300	
10	....	556
	500	
9	....	1056
Total	1056 c.y.	
½ total	528	

The tabulation shows that in passing from 12 + 28 to 11 the total intervening volume is 256 which is less than 528 (the half-total volume); and that in passing from 12 + 28 to 10 the total intervening volume is 556 which is greater than the 528. We pass 300 c.y. in moving over the 100 ft. between 11 and 10, or at the rate of $\frac{300}{100} = 3$ c.y. per running foot (assuming the prismoid 11–10 to be of uniform cross-sectional area). To come to the center of the mass of the series of prismoids we must move from 11 toward 10 a distance of $\frac{(528 - 256)}{3} = 90.7$ ft., which brings us to sta. 11 − 90.7 = sta. 10 + 09.3. Thus the same result is obtained whether we start from one end or the other. In practice the computations should be made from one end and then the result checked by computing from the other.

23. Center of mass of a series of prismoids. Two mass-curve solutions. — (a) *Common mass-curve solution.* — Using the data shown in Fig. 21 which represents a profile, we proceed as follows: (1) Construct the mass curve (Secs. 10 and 20) for the station-volumes lying between stas. 9 and 12 + 28. The maximum ordinate is $C'C$. (2) Mark the point C'' which bisects the ordinate $C'C$. (3) Through C'' draw a horizontal line cutting the mass curve AC at some point M. (4) The station and plus of M is now read as sta. 10 + 10, which is the center of mass of the series of prismoids. This result differs by 0.7 ft. from that found in the preceding section.

As the horizontal scale of profile paper is 1 in. = 400 ft., the uncertainty in the position of the center of mass, as found by the use of the mass curve, is, in general, not less than 4 ft. (Sec. 16). The results found by the algebraic and graphical methods in any case should substantially agree.

(b) *Another mass-curve solution.* — The following method of finding the center of mass is taken from a letter by Mr. T. S. Russell to *Engineering News*. The letter appeared in the issue of March 14, 1891, under the caption "The Calculation of Overhaul"; was reprinted in Gillette's "Earthwork and Its Cost," pp. 214–217; and reprinted again in Proceedings of American Railway Engineering Association, vol. 7 (1906), pp. 386–389.

The profile and "first" mass curve of Fig. 21a are the same as the profile and mass curve of Fig. 21. Now, in Fig. 21a, plot a "second" mass curve, $C'MA'$, for the same cut *ac*, taking the origin at C' and plotting from right to left. The ordinate at sta. 12 is 56 c.y.; at sta. 11, 56 + 200 = 256 c.y.; at sta. 10, 256 + 300 = 556 c.y.; at sta. 9, 556 + 500 = 1056 c.y. Call the point of intersection of the first and second mass curves, *M*. The vertical *Mm* contains the center of mass of the cut *ac*. By scaling, *M* is found to be 10 ft. to the right of sta. 10; hence, the center of mass of cut *ac* lies at sta. 10 + 10. This is the same as the result found in Sec. 23; though, owing to errors of plotting and scaling, such agreement is not to be regularly expected. See Secs. 16, 17.

CHAPTER IV

CENTER OF GRAVITY

In this chapter are described eight methods (so-called) of finding the center of gravity of a body of material, ranging from the roughly approximate to the practically exact, and using the eye, arithmetic, or the mass curve. All the methods are applied to the same problem, namely, finding the center of gravity of the cut between 9 + 00 and 12 + 28 of Fig. 19. (Figs. 19, 21, 21a, and 24 face p. 96.)

24. **Relation between haul and center of gravity.** — We have seen (Sec. 1) that one of the two factors of haul is the distance between the center of gravity of a body of material in cut and the center of gravity of the same body of material in fill. To find the distance factor we shall first find the position of each center of gravity. Methods of finding the position of the center of gravity of a body of material are presented in the following sections.

25. **Center of gravity of single prismoid.** — In most cases it serves every requirement to assume that the center of gravity lies at the center of length of the prismoid. For example, the center of gravity of the prismoid lying between stas. 12 and 13 (Fig. 22) may be assumed to be at 12 + 50. In rare cases it may be thought that the foregoing assumption will not give satisfactory results, and in such cases the true center of gravity of each single prismoid (station- or substation-volume) is determined in the following manner.

Let A_{12} = cross-sectional area at sta. 12;
A_{13} = cross-sectional area at sta. 13.

It is plain that the true center of gravity of prismoid 12–13 lies between sta. 12 + 50 and the station having the larger area. Thus, as A_{12} is greater than A_{13} (as shown in the figure), the center of gravity lies to the left of sta. 12 + 50.

Let x = the horizontal distance between the true center of gravity and the mid-point of the prismoid;

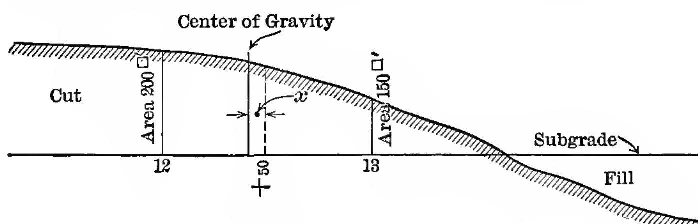
l = station-interval (length of prismoid);

V = volume (c.y.) of prismoid.

Then,

$$x = \frac{l^2}{12}\left(\frac{A_n - A_{n-1}}{27V}\right), \tag{21}$$

where A_n and A_{n-1} are the cross-sectional areas at the two adjacent stations. (See Allen's "Railroad Curves and Earthwork," pp. 192–193, for derivation of this formula, and Raymond's "Railroad Field Geometry," p. 222, eq. 225.)



CENTER OF GRAVITY OF SINGLE PRISMOID

FIG. 22.

If we substitute for V in eq. 21 its value $\frac{A_n + A_{n-1}}{2 \times 27} l$, we have

$$x = \frac{l}{6}\left(\frac{A_n - A_{n-1}}{A_n + A_{n-1}}\right). \tag{22}$$

(See Searles' "Engineers' Field Book," p. 244, eq. 361.)

Example. — In Fig. 22, A_{12} = 200 sq. ft., and A_{13} = 150 sq. ft. Length of prismoid 12–13 is 100 ft. Therefore the center of gravity of the prismoid lies to one side of the mid-section the distance

$$x = \frac{100}{6}\left(\frac{200 - 150}{200 + 150}\right) = 2.4 \text{ ft.};$$

and since the area at sta. 12 is greater than the area at sta. 13, the center of gravity lies on the left of the mid-section. Hence the center of gravity lies at (sta. 12 + 50) − 2.4 ft. = sta. 12 + 47.6.

For the error in overhaul resulting from the use of the center of length instead of the center of gravity of the prismoid, see Sec. 146.

26. Center of gravity of a series of prismoids. — Eight methods of finding the center of gravity of a series of prismoids will be given ranging from roughly approximate to closely approximate. The methods will be illustrated, in the following sections, by application to one of the profiles of Figs. 19, 21, 24. See columns 6 and 10, Fig. 36, for center of gravity determined by each of the eight methods.

27. Method I. Center of gravity determined by eye. — This method serves for rough estimates and rough check on results obtained by other methods. The center of gravity of the cut (Fig. 19) between sta. 9 and sta. 12 + 28 appears to be at about sta. 10 + 60. The person who finds the center of gravity by inspection of the profile must be ignorant of the computed center of gravity; otherwise his result will be biased and of no value.

28. Method II. Center of gravity assumed to lie at the center of length. — When the profile of the cut (or fill) is practically symmetrical about a central vertical line, the center of gravity may, for rough estimates and checks, be assumed to lie at the center of length. Though the cut, sta. 9 to sta. 12 + 28 (Fig. 19) lacks symmetry, we use it for an example for this method. The center of length lies at sta. 9 + $\frac{1}{2}$ [(sta. 12 + 28) − (sta. 9 + 00)] = sta. 9 + 164 = sta. 10 + 64.

29. Method III. Series of prismoids treated as a single prismoid. When the area of cross-section increases (or decreases) with practical regularity as we pass from the initial to the final station of the series of prismoids, a rough determination of the position of the center of gravity of the series can be made by considering the series as a single prismoid and applying to this prismoid the method given in Sec. 25 (center of gravity of single prismoid).

Apply this method to the series of prismoids (Fig. 19) lying between stas. 9 and 12 + 28. The mid-point of the series is at sta. $9 + \frac{1}{2}$ [(sta. 12 + 28) − (sta. 9 + 00)] = sta. 9 + 164 = sta. 10 + 64. The area of the section at sta. 9 is 162 sq. ft.; the area of the section at sta. 12 + 28 is 54 sq. ft. The length of the prismoid is here (sta. 12 + 28) − (sta. 9 + 00) = 328 ft. By eq. 22, the distance in ft. from the center of gravity of the prismoid to its center of length is

$$x = \frac{\text{length (ft.)}}{6} \times \frac{\text{difference between end areas (sq. ft.)}}{\text{sum of end areas (sq. ft.)}}$$

$$= \frac{328}{6} \times \frac{162 - 54}{162 + 54} = 27.3 \text{ ft.}$$

So the center of gravity lies 27.3 ft. to the left of the mid-point (sta. 10 + 64) of the prismoid: at sta. 10 + 36.7.

30. Method IV. Center of gravity assumed to lie at center of mass. Arithmetical solution. — This approximate method, frequently used, even in computing pay quantities, is illustrated in Sec. 22 by the use of data given in Fig. 19. It was found in Sec. 22 that the center of mass of the cut, 9 + 00 to 12 + 28, lies at sta. 10 + 09.3.

31. Method V. Center of gravity assumed to lie at center of mass. Mass-curve solution. — Two ways of finding the center of gravity come under this head. Both ways are in use even for the computation of pay quantities. Both are given here although only the first will be used in the examples of Chapters VI and VII.

(a) *Common mass-curve solution.* — This method of finding the center of gravity of a series of prismoids is explained in Sec. 23 (a) and it was there found that the center of mass (here assumed to coincide with the center of gravity) of the cut, 9 + 00 to 12 + 28, lies at 10 + 10.

(b) *Another mass-curve solution.* — This method is the same as the foregoing except in the detail of the use of the mass curve, and is explained in Sec. 23 (b), and illustrated in Fig. 21a.

The center of mass coincides with the true center of gravity of a body of material when the mass curve is straight between the terminal stations of that body.

32. **Method VI. Center of gravity of each prismoid assumed to lie at its mid-point. Arithmetical solution.** — This method, one of the standard methods used in computing pay quantities, is as follows: Fig. 23. Let *m*, *n*, *o*, *p* be consecutive stations

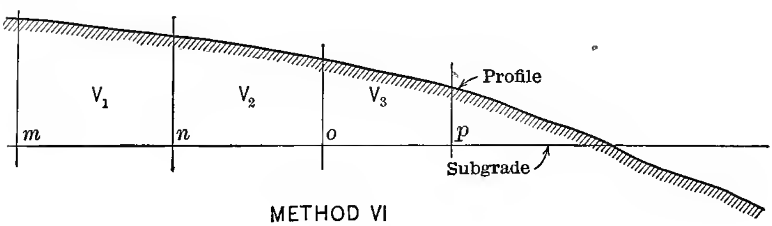


FIG. 23.

(or substations) along the profile. Let the center of gravity of each prismoid be assumed to lie at its center of length (except for this assumption, this method is exact). Let V_1, V_2, V_3, . . . be the respective volumes (c.y.) of the prismoids, *mn*, *no*, *op*, . . .

Taking *m* as a center of moments:

The moment of V_1 about *m* is $V_1 \frac{mn}{2}$.

The moment of V_2 about *m* is $V_2 \left(mn + \frac{no}{2}\right)$.

The moment of V_3 about *m* is $V_3 \left(mn + no + \frac{op}{2}\right)$.

The distance (stas.) from the center of moments (*m*, in this case) to the center of gravity of the series is

$$\frac{\text{sum of the moments about } m \text{ (sta-yds.)}}{\text{sum of the volumes (c.y.)}}. \qquad (23)$$

Applying this method to the cut between stas. 9 and 12 + 28 (Fig. 19) we make the computations below.

MOMENTS ABOUT STATION 9

Station.	Station of center of length (assumed to be center gravity of prismoid).	V = volume (c.y.).	a = lever-arm (stas.).	Va = moment (sta-yds.).
9				
10	9+50	500	.50	250
11	10+50	300	1.50	450
12	11+50	200	2.50	500
12+28	12+14	56	3.14	176
	Totals.......	1056 c.y.	....	1376 sta-yds.

Average lever-arm = distance from sta. 9 to center of gravity of series of prismoids $\Big\} = \frac{1376}{1056} =$ 1.303 stas.

Hence station of center of gravity = (sta. 9 + 00) + 1.303 stas. = sta. 10 + 30.3.

As a check on the computations above we may find the center of gravity by taking moments about the other extreme station, 12 + 28.

MOMENTS ABOUT STATION 12 + 28

Station.	Station of center of length (assumed to be center gravity of prismoid).	V = volume (c.y.).	a = lever-arm (stas.).	Va = moment (sta-yds.).
12+28				
12	12+14	56	.14	8
11	11+50	200	.78	156
10	10+50	300	1.78	534
9	9+50	500	2.78	1390
	Totals.......	1056 c.y.	....	2088 sta-yds.

Average lever-arm = distance from sta. 12 + 28 to center of gravity of series of prismoids lying between stas. 12 + 28 and 9 + 00. $\Bigg\} = \frac{2088}{1056} =$ 1.977 stas.

Hence station of center of gravity is (sta. 12 + 28) − 1.977 stas. = sta. 10 + 30.3 (check).

Note. — The two tabular forms above may be combined in one with saving of space and time. See Sec. 118.

33. Use either measured or equated fill-volumes in computing by moments (Methods VI and VIII, Secs. 32 and 35) the station of center of gravity of series of fill prismoids. — It matters not in finding center of gravity of a fill by Method VI, whether we use the fill-volumes or the equated fill-volumes as factors in computing moments, provided the equating-factor be constant.

Let $V_1,\ V_2,\ V_3$ = the respective volumes of fill;

q = constant equating-factor;

then $V_1q,\ V_2q,\ V_3q$ = the respective equated volumes of fill.

Let $a_1,\ a_2,\ a_3$ = respective lever-arms of vols. $V_1,\ V_2,\ V_3$.

By using fill-volumes $V_1,\ V_2,\ V_3$,

$$\text{the average lever-arm} = \frac{V_1a_1 + V_2a_2 + V_3a_3}{V_1 + V_2 + V_3}, \tag{24}$$

while, by using equated fill-volumes $V_1q,\ V_2q,\ V_3q$,

$$\begin{aligned}\text{the average lever-arm} &= \frac{V_1qa_1 + V_2qa_2 + V_3qa_3}{V_1q + V_2q + V_3q} \\ &= \frac{q}{q}\left(\frac{V_1a_1 + V_2a_2 + V_3a_3}{V_1 + V_2 + V_3}\right) \\ &= \frac{V_1a_1 + V_2a_2 + V_3a_3}{V_1 + V_2 + V_3}. \end{aligned} \tag{24a}$$

The average lever-arm is thus the same whether we compute it by use of measured fill-volumes or equated fill-volumes.

34. Method VII. Center of gravity of each prismoid assumed to lie at its mid-point. Mass-curve solution.—This is the same as Method VI except that here the work is carried out graphically instead of arithmetically. We present this method by application to Fig. 24, which shows part of the cut of Fig. 19.

(1) Draw the mass curve AC (Fig. 24).

(2) Find the area (sq. in.) ACJ lying between the mass curve and its projection on the vertical through A (sta. 9). The area may be found by means of the planimeter or otherwise. We find the area to be 1.38 sq. in. when measured by the planimeter.

(3) Multiply the area ACJ (sq. in.) by the sta-yds. represented by 1 sq. in. of paper (Sec. 14), obtaining the total haul H. $H = 1000 \times 1.38 = 1380$ sta-yds.

(4) Divide H by total volume (1064,* obtained by scaling ordinate CC') of prismoids lying between stas. 9 and 12 + 29,† obtaining the distance from sta. 9 out to the center of gravity of the series of prismoids. Distance $= \frac{1380}{1064} = 1.298$ stas. Hence the center of gravity of the series of prismoids is at (sta. 9 + 00) + 1.298 stas. = sta. 10 + 29.8.

Otherwise:

(1) Draw the mass curve AC.

(2) Find area ACJ in square inches.

(3) Divide ACJ by CC' (inches), and multiply by the number of horizontal feet per inch of paper, thus obtaining the distance from sta. 9 to the center of gravity of the volume lying between stas. 9 and 12 + 29.

The same result would be obtained if we dealt with the area ACC' instead of the area ACJ, except that with the area ACC' the resulting distance would be that between sta. 12 + 29 and the center of gravity of the series of prismoids lying between stas. 12 + 29 and 9. Thus the planimeter gives the area $ACC' = 2.1312$ sq. in. Therefore $H = 2.1312 \times 500 \times 2.00 = 2131$ sta-yds. Hence the distance from 12 + 29 to the center of gravity of the series of prismoids $= \frac{2131}{1064} = 2.003$ stas. Hence the center of gravity of the series of prismoids lies at (sta. 12 + 29) − 2.003 stas. = 10 + 28.7. The discrepancy between the

* The practicable way of determining volume, when using a given mass curve, is to scale an ordinate; and for this reason we have here used the value 1064 c.y. of CC' determined by scaling, notwithstanding we cannot have forgotten that CC' was plotted to represent 1056 c.y.

† The point c, Fig. 24, is the left-hand limit of the free-haul distance shown in Fig. 19. In Sec. 40 it is found by arithmetic that c lies at sta. 12 + 28; while in Sec. 41 it is found, by the use of the mass curve, that C lies at sta. 12 + 29; and this accounts for the fact that sometimes we speak of the cut 9 + 00 to 12 + 28, and at other times of cut 9 + 00 to 12 + 29.

two results comes from the errors involved in taking off the areas. The larger the scale of ordinates for the mass curve, the smaller will be the discrepancy. See Secs. 16, 17, 18.

35. **Method VIII. The true center of gravity.**—This method is the same as Method VI except that in the former we locate the true center of gravity of each prismoid by the method given in Sec. 25; whereas in the latter we took the mid-point of each prismoid for its center of gravity.

From Sec. 25, eq. 22, the distance in feet between the mid-point and the center of gravity of a prismoid is

$$x = \frac{\text{length (ft.)}}{6}\left(\frac{\text{difference between end areas (sq. ft.)}}{\text{sum of end areas (sq. ft.)}}\right).$$

To illustrate, let us apply this method to the series of prismoids lying between stas. 9 and 12 + 28 of Fig. 19.

COMPUTATION FOR POSITION OF CENTER OF GRAVITY OF EACH PRISMOID

Station.	l = length of prismoid (ft.).	A = end area (sq. ft.)	Diff. between end areas.	Sum of end areas.	$\frac{\text{Diff.}}{\text{Sum}}$.	$\frac{l}{6} = \frac{\text{length}}{6}$.	x	Station of center of prismoid.	Station of center of gravity.
9	...	162	..	...	...	.....	..	......	....
	100	...	54	270	0.2	16.7	3.3	9+50	9+46.7
10	...	108	..	...	...	....	...	......	....
	100	...	54	162	0.3	16.7	5.0	10+50	10+45
11	...	54	..	...	...	....	...	......	....
	100	...	0	108	0.0	16.7	0.0	11+50	11+50
12	...	54	..	...	...	....	...	......	....
	28	...	0	108	0.0	4.7	0.0	12+14	12+14
12+28	...	54	..	...	...	....	...	......	....

NOTE.—To compute the stations of the centers of gravity of the prismoids, is over and above the work required in the less exact Method VI.

MOMENTS ABOUT STATION 9 + 00

Station.	Station of center of gravity.	V = vol. of prismoid (c.y.).	a = lever-arm about station 9 (stas.).	Va = moment (sta-yds.).
9				
10	9+46.7	500	.467	233
11	10+45	300	1.450	435
12	11+50	200	2.50	500
12+28	12+14	56	3.14	176
	Totals.....	1056 c.y.	....	1344 sta-yds.

The average lever-arm = distance of center of gravity of the series of prismoids from sta. 9 + 00. $= \frac{1344}{1056} = 1.273$ stas.

Hence, center of gravity of the series of prismoids lies at sta. 9 + 127.3 = sta. 10 + 27.3.

Check this result by taking moments about 12 + 28.

MOMENTS ABOUT STATION 12 + 28

Station.	Station of center of gravity.	V = vol. of prismoid (c.y.).	a = lever-arm about sta. 12 + 28 (stas.).	Va = moment (sta-yds.).
12+28				
12	12+14	56	.14	8
11	11+50	200	.78	156
10	10+45	300	1.83	549
9	9+46.7	500	2.813	1406
	Totals.....	1056 c.y.	....	2119 sta-yds.

The average lever-arm = distance from center of gravity of the series of prismoids to sta. 12 + 28. $= \frac{2119}{1056} = 2.007$ stas.

Hence the center of gravity of the series of prismoids is at (sta. 12 + 28) − 2.007 stas. = sta. 10 + 27.3 (check).

CHAPTER V

OVERHAUL, FREE HAUL, AND CROSSHAUL

In this chapter, overhaul is defined; a digest of American practice in computing overhaul is given; the American Railway Engineering Association's specification, which is used as the basis for all the problems worked in this book, is presented; and three methods — by eye, by arithmetic, and by mass curve — of determining free-haul limits are explained. Crosshaul is discussed; and eight methods of computing overhaul are characterized. (Fig. 19 faces p. 96; Fig. 36, p. 67; Fig. 38, p. 120.)

36. **Definitions.** — In some contracts for railroad grading it is stipulated that the contractor shall be paid so much per cubic yard for excavating, hauling, and dumping, regardless of the distance which the material is hauled; in others, that a certain price shall be paid per cubic yard for excavating, hauling, and dumping, for all material hauled less than a specified distance, and that an extra sum shall be paid for each cubic yard which is hauled a distance in excess of the specified distance. It is specified that to determine the number of cubic yards of material excavated, hauled, and dumped, the material shall be measured in its original condition; in other words, payment is on the basis of cubic yards of material in place.

The "specified distance" above is called the free-haul distance. The "distance in excess of the specified distance" above is called the overhaul distance. The "extra sum" above is the money paid for hauling a cubic yard the overhaul distance. By overhaul price is usually meant the price stipulated for hauling one cubic yard a distance of one station (100 ft.). Overhaul is the product of the number of cubic yards hauled, by the average overhaul-distance. The overhaul distance is usually expressed in stations of 100 ft., and accordingly the overhaul is expressed in station-yards. See Sec. 1.

37. Basis of overhaul computation: review of current practice in America. — The subject of overhaul has received extended consideration by the American Railway Engineering Association which presents in its Proceedings, vol. 7 (1906), more than one hundred pages (pp. 341 to 449) devoted to discussions on overhaul, a "Review of the Literature of Overhaul," and an "Abstract of Current Practice of Association Members." This Abstract was compiled from replies to the following circular letter sent by the Association to its members in 1905:

> The Committee on Roadway desires an expression from you as to your practice relative to paying for overhaul; your usual limit of free haul [i.e., free-haul distance]; and your method [i.e., basis] of computing overhaul.
>
> Please indicate on the enclosed blank form which of the following methods [i.e., bases] described by Mr. Walter G. Berg in vol. 4 of the Proceedings for 1903, pp. 118–124, you prefer:
>
> (A) All cuts of the section are taken into account, the average haul [distance] of each individual mass determined, and the average total haul [distance] established accordingly. The free haul [distance] is deducted from the total average haul [distance] and the balance represents the overhaul [distance] applicable to the total yardage of excavation of this section.
>
> This is called the free average haul [distance] method [i.e., basis] applicable to all excavation of the section. [See Sec. 39 for an interpretation of basis (A).]
>
> (B) All cuts, the material from which is disposed of within the free-haul limit[s], are left out of consideration entirely. All other cuts from which the material, or at least some of the material, is hauled beyond the free-haul limit [i.e., free-haul distance] are taken into account, the average haul [distance] of each individual mass of same determined and the average total haul [distance] established accordingly. The free haul [distance] is deducted from the average total haul [distance], and the balance represents the overhaul [distance] applicable to the total yardage of excavation of such cuts only from which the material

or some of the material is hauled beyond the free limit.

This method [i.e., basis] is called the free average haul, applicable, however, only to cuts with overhauls. [See Sec. 39 for an interpretation of basis (B).]

(C) All cuts, the material from which is disposed of within the free-haul limit[s] are left out of consideration entirely. At all other cuts the material in the cut required to balance the material in the nearest fill or fills is determined within the free-haul limit[s]; in other words, a strip equal to the free-haul limit [i.e., free-haul distance] is cut out of the profile in such a way that the cut balances the fill within the limit[s] of this strip. This balanced material within the free-haul strip is not taken into account. All other material that is clearly hauled more than the free-haul limit [i.e., free-haul distance] is taken into account, the average haul [distance] of each individual mass determined, and the average total haul [distance] established accordingly. The free-haul [distance] is deducted from the average total haul [distance], and the balance represents the overhaul [distance] applicable to the total yardage of such parts of cuts as are hauled beyond the free limit[s].

This method [i.e., basis] is called the free straight haul, as it eliminates consideration of absolutely all material that is hauled less than the free limit [i.e., free haul distance]. [Basis (C) is used throughout the following pages of this book. See Sec. 38.]

Replies to the circular letter above were received from 124 members of the Association, connected with 75 railroads. Basis (A) was preferred by 4 members; basis (B), by 21; basis (C), by 54; and 4 members preferred other bases; while 37 members omitted the overhaul clause from the contract.

As to the free-haul distance: 32 members preferred 500 ft.; 16, 1000 ft.; 4, 400 ft.; 4, 300 ft.; 3, 700 ft.; 3, 600 ft.; 3, 200 ft.; 2, 100 ft.; 2, 200–500 ft.; 2, 200–1000 ft.; 2, 500–1000 ft.; one each, 150 meters; 250 meters; 800 ft.; 1500 ft.; and 500–1200 ft. See Proceedings, American Railway Engineering

Association, vol. 7 (1906), pp. 429–431, from which the foregoing facts are taken.

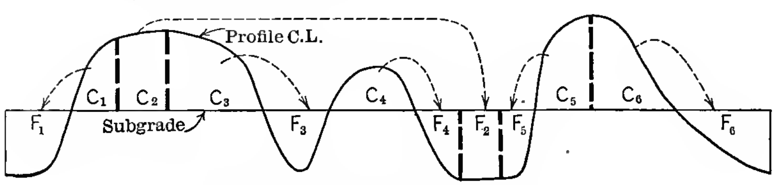
38. **Basis of overhaul computation recommended by American Railway Engineering Association.** — In June, 1906, the Association adopted by letter ballot the following alternate optional overhaul-clause which is virtually basis (C) presented above. On pp. 26 and 27 of the Manual (edition of 1911) of the Association will be found, incorporated in "Specifications for the Formation of the Roadway," the following clauses on overhaul:

> 48. Unless otherwise specified, it is distinctly understood that the contract price per cubic yard covers any haul found necessary, and that there shall be no allowance made for any so-termed overhaul.

Alternate Optional Overhaul Clause

> (*The following alternate optional overhaul clause is recommended to be substituted for clause No. 48 of the Specifications for the Formation of the Roadway in all cases where it is desired to allow overhaul.*)
>
> 48-a. No payment shall be made for hauling material when the length of haul does not exceed the limit of free haul [i.e., free-haul distance], which shall be   ft.
>
> The limits of free haul shall be determined by fixing on the profile two points — one on each side of the neutral grade point — one in excavation and the other in embankment, such that the distance between them shall equal the specified free-haul limit [i.e., free-haul distance] and the included quantities of excavation and embankment balance. All haul on material beyond this free-haul limit [i.e., free-haul distance] shall be estimated and paid for on the basis of the following method of computation, viz.:
>
> All material within the limit[s] of this free haul shall be eliminated from further consideration.
>
> The distance between the center of gravity of the remaining mass of excavation and center of gravity of the resulting embankment, less the limit of free-haul [i.e., less the free-haul distance], as above specified, shall be the length of

> overhaul [i.e., the overhaul distance]; and the compensation to be rendered therefor shall be determined by multiplying the yardage in the remaining mass, as above described, by the length of the overhaul [i.e., by the overhaul distance]. Payment of the same shall be by units of one cubic yard hauled one hundred (100) feet.
>
> Where material is obtained from borrow pits alongside the embankment and runways are constructed, the haul [distance] shall be determined by the distance the team necessarily travels. The overhaul on material thus hauled shall be determined by multiplying the yardage so hauled by one-half the round distance made by the team less the free-haul distance. The runways shall be established by the engineer.



OVERHAUL: BASIS A AND BASIS B

FIG. 25.

It will be seen that the overhaul clause above is basis (C) given in Sec. 37.

All directions given in this book for the computation of overhaul assume that the overhaul clause "48a" next above is the *basis* of computation.

For the various *methods* of computing overhaul on this American Railway Engineering Association basis, see Sec. 47.

39. Interpretation of basis (A) and basis (B). — Although neither basis (A) nor (B) of Sec. 37 will be used in this book, the following interpretation is given to make clear the differences between the three bases cited and to enable the inexperienced computer to compute overhaul by either (A) or (B) if that is desired.

Let us apply basis (A) for the overhaul of Fig. 25, in which the arrows indicate the distribution of the material. It is assumed that cut C_1 precisely makes the fill F_1; cut C_2 the fill F_2; and so on.

Let $C_1, C_2, \ldots$ = volumes (c.y.) of bodies $C_1, C_2, \ldots,$ respectively, of cut;

h_1 = distance (in stations) between the center of gravity of cut C_1 and center of gravity of fill F_1;

h_2 = distance (in stations) between the center of gravity of cut C_2 and center of gravity of fill F_2, and so on;

f = free-haul distance in stations;

O_A = overhaul (sta-yds.) computed on basis (A);

O_B = overhaul (sta-yds.) computed on basis (B).

Then according to the rules under basis (A) (Sec. 37),

$$O_A = \left(\frac{C_1h_1 + C_2h_2 + \cdots + C_6h_6}{C_1 + C_2 + \cdots + C_6} - f\right)(C_1 + C_2 + \cdots + C_6). \quad (25)$$

If some of the material of every cut is hauled farther than the free-haul distance, the overhaul will be the same when computed on basis (B) as when computed on basis (A). But suppose that all of the material of cuts C_4 and C_5–C_6 is disposed of within the free-haul distance; then the overhaul computed on basis (A) will be the same as before, but computed on basis (B) will be

$$O_B = \left(\frac{C_1h_1 + C_2h_2 + C_3h_3}{C_1 + C_2 + C_3} - f\right)(C_1 + C_2 + C_3). \quad (26)$$

Observe that if one portion of a cut, as portion C_2 of cut C_1–C_2–C_3 (Fig. 25), is hauled farther than the free-haul distance, all other portions of the cut come into the computation whether they are hauled farther than the free-haul distance or not.

40. Free-haul limits: arithmetical method. — This method is presented by application to a particular case. In Fig. 19 the free-haul distance is assumed to be 300 ft. The swell ratio is assumed to be 0.25. To fix the free-haul limits in this case we take the following steps:

1. Equate the fill-volumes (given in Fig. 19) to volumes in place (Sec. 7). (The equated fill-volumes are entered on the profile.) In the succeeding steps we use the equated fill-volumes along with the cut-volumes.

2. We take stas. 12 and 15 as trial free-haul limits. This gives $80 + 200 = 280$ c.y. of cut and $20 + 120 = 140$ c.y. of fill. Therefore between stas. 12 and 15, cut exceeds fill by $280 - 140 = 140$ c.y.; and we must shift the 300-ft. free-haul distance to the right (toward the fill) some distance, x, say, in order to decrease the cut and increase the fill to a point where the cut and fill between the limits just balance.

3. The volume per running foot between stas. 12 and 13 is $\frac{200}{100} = 2$ c.y. (average); and between stas. 15 and 16 is $\frac{300}{100} = 3$ c.y. (average). Therefore if we shift the 300-ft. free-haul distance one foot to the right we decrease the 280 c.y. of cut by 2 c.y., and increase the 140 c.y. of fill by 3 c.y. Hence for each foot of shift toward the right the difference, 140 c.y., between cut- and fill-volumes is decreased by $2 + 3 = 5$ c.y. Therefore to reduce the difference to zero — to make the cut and fill balance — we must shift the 300-ft. free-haul distance to the right a distance equal to $x = \frac{140}{5} = 28$ ft. Thus we find that the free-haul limits are at stas. 12 + 28 and 15 + 28.

If the trial limits had been taken as stas. 13 and 16 we should have reached the same result. In such case we should have: Cut = 80 c.y.; fill $= 20 + 120 + 300 = 440$ c.y. Fill is in excess by $440 - 80 = 360$ c.y. Therefore we shift the free-haul distance to the left a distance $x' = \frac{360}{5} = 72$ ft.; and this places the free-haul limits finally at stas. 12 + 28 and 15 + 28, as before.

41. Free-haul limits: mass-curve method. — This graphical method of fixing upon the limits of free haul is always used when the mass curve is used for computing overhaul; and may be used when the overhaul is computed arithmetically. In the following it is assumed that the mass curve is to serve no purpose other than finding the free-haul limits.

Below the profile, Fig. 26, draw a base line below which to lay off mass-curve ordinates. To a convenient scale lay off below

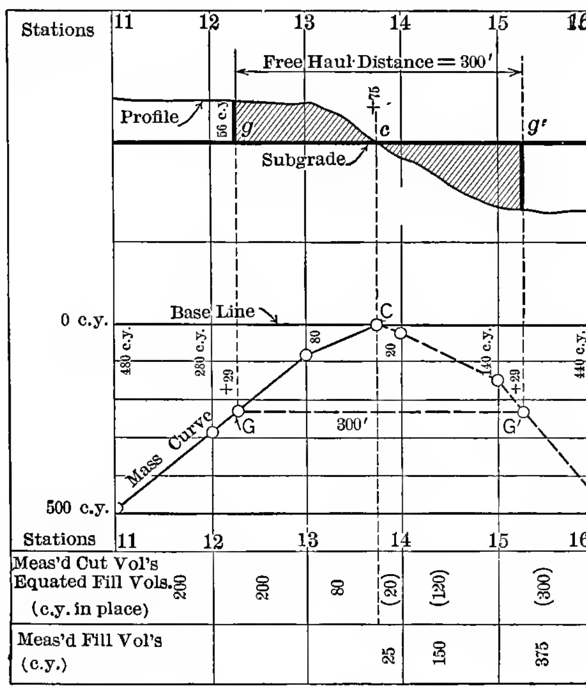


FIG. 26.—Limits of Free Haul.

the base line at each station in cut the total cubic yards of cut lying between that station and the grade point, *c*, of the profile. Connect the plotted points, two and two, by straight lines, thus forming a mass curve with origin at the grade point. In like manner lay off below the base line at each station in fill the total cubic yards of equated fill lying between that station and the grade point, *c*, of the profile. Draw a broken line through the points thus plotted. (The mass curve need not be carried farther to the right or the left of the grade point than the free-haul distance.)

Now find two points, *G* and *G′*, the one on the left branch and the other on the right branch, of the mass curve which lie on the same horizontal line and are separated by a distance equal to the free-haul distance. One way of fixing the positions of *G* and *G′* is to place the zero of the engineers' scale on the left branch of the curve while the edge of the scale is parallel to the base line (that is, horizontal); then slide the scale upward or downward, all the time keeping the zero of the scale on the left branch of the mass curve and maintaining the horizontality of the scale, until the scratch on the scale marking the free-haul distance comes to lie precisely on the right branch of the curve. Draw a line along the edge of the scale in this position cutting the mass curve at *G* and *G′*. *G* and *G′* are the limits of free haul because the ordinates at *G* and *G′* are equal, showing that between them the cut will just make the fill, and also the distance between the points is the free-haul distance.

Having determined the points *G* and *G′* on the paper, it remains to read the station and plus of each and enter the plus on the profile. Thus we find that *G* is at sta. 12 + 29, and *G′* is at sta. 15 + 29 (see last part of Sec. 129).

If the mass curve has been drawn for the whole stretch of profile under consideration in connection with an overhaul problem, the free-haul limits are determined in the manner shown in Fig. 19 and Sec. 129.

NOTE. — In Fig. 27 let it be assumed that all the material of cut *bc* is needed for the fill or fills to the right and that none of the material of cut *bc* is needed to the left. In this case it is proper to fix the free-haul limits *xy* on the right end of the cut, and on that end only. The material *xc* is assumed to be placed in the fill *cy* whether it is actually so placed or not. See Sec. 3, second paragraph from the end, and Sec. 44, last paragraph.

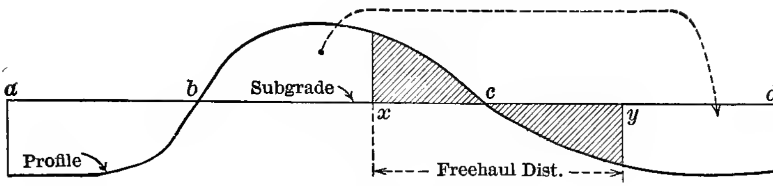
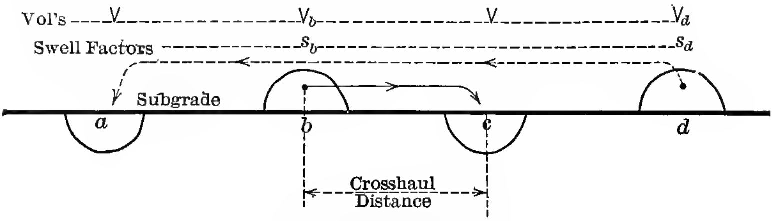


FIG. 27.

42. Free-haul limits determined by eye. — Sometimes, as when a rough estimate of haul or overhaul must be made on short notice, the free-haul limits are determined directly on the profile by eye. This may be done by taking a space on the engineers' scale representing the free-haul distance, and so placing the scale by trial that between the two extremities of this space the cut appears to balance the fill, and then marking the profile paper at the extremities of the free-haul space. The two marks are the approximate limits of free haul. A better way when this operation is to be several times repeated is this: Cut out of a piece of drawing paper a rectangular frame making the length of the opening equal (to the horizontal scale of the profile) to the free-haul distance. The frame should be an inch or more wide at each end. By applying the frame instead of the scale, the cut areas and fill areas between the trial limits of free haul are more quickly and more accurately compared by eye. See Sec. 61 for the application of this method of determining limits of free haul, to the profile of Fig. 19.

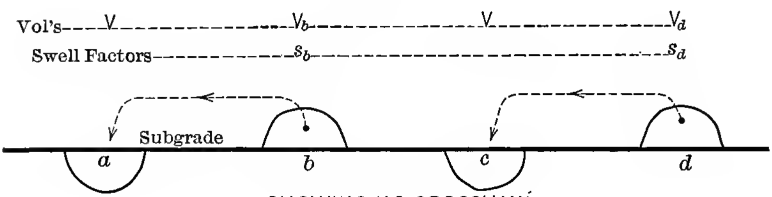
It should be remembered when using this method of determining the limits of free haul, that the volume of a cut or a fill per running foot is proportional to neither the center height nor the area of the profile. The estimate, to be of much value, must be made by one who is ignorant of limits which may have been computed.

43. Crosshaul and its effect on total haul. — Crosshaul occurs when in the work of grading, excavated material is carried in both directions over the same identical part of the line. The nature of crosshaul is clearly shown in Figs. 28 to 31.



SHOWING CROSSHAUL

FIG. 28.



SHOWING NO CROSSHAUL

FIG. 29.

Let it be assumed that the volume of fill a = volume of fill $c = V$ c.y. Let s_b be the swell-factor of cut b and s_d be the swell-factor of cut d. Then it will take $\frac{V}{s_b}$ c.y. from cut b to make V c.y. of fill and $\frac{V}{s_d}$ c.y. from cut d to make V c.y. of fill.

Assume that there are $V_b = \frac{V}{s_b}$ c.y. of material in cut b and $V_d = \frac{V}{s_d}$ c.y. of material in cut d.

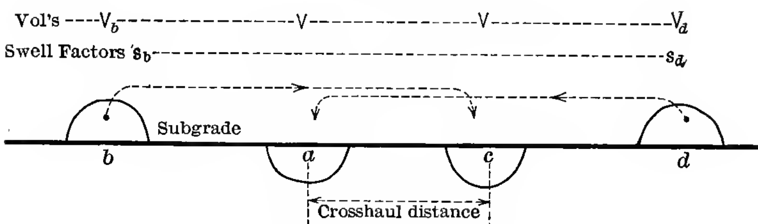
Now if, as shown by arrows in Fig. 28, V_b c.y. is hauled from b to c, and V_d c.y. is hauled from d to a, the resulting haul is

$$H_{28} = V_b bc + V_d da$$

$$= V\left(\frac{bc}{s_b} + \frac{ab}{s_d} + \frac{bc}{s_d} + \frac{cd}{s_d}\right). \qquad (27)$$

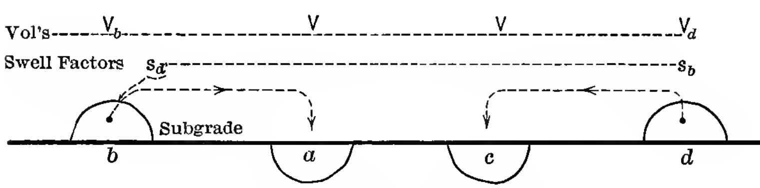
If, on the contrary, V_b is hauled from b to a, and V_d is hauled from d to c, as indicated by the arrows in Fig. 29, the resulting haul is

$$H_{29} = V_b ab + V_d cd$$
$$= V\left(\frac{ab}{s_b} + \frac{cd}{s_d}\right). \tag{28}$$



SHOWING CROSSHAUL

FIG. 30.



SHOWING NO CROSSHAUL

FIG. 31.

Hence, the excess haul due to the crosshaul is

$$H_{28} - H_{29} = V\left(\frac{bc}{s_b} + \frac{ab}{s_d} + \frac{bc}{s_d} + \frac{cd}{s_d}\right)$$
$$- V\left(\frac{ab}{s_b} + \frac{cd}{s_d}\right)$$
$$= V\left(bc\left(\frac{1}{s_b} + \frac{1}{s_d}\right) + ab\left(\frac{1}{s_d} - \frac{1}{s_b}\right)\right). \tag{29}$$

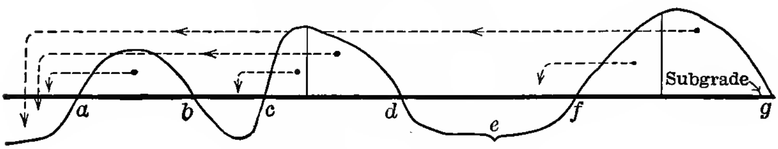
It is evident that $H_{28} - H_{29}$ may in some cases be negative, that is, under some conditions of distance and swell, crosshauling may actually result in minimum haul.

For the special case in which the two swell-factors are equal, so that $\frac{1}{s_b} = \frac{1}{s_d} = \frac{1}{s}$, say, eq. 29 reduces to

$$H_{28} - H_{29} = V\left(\frac{2\,bc}{s}\right). \qquad (30)$$

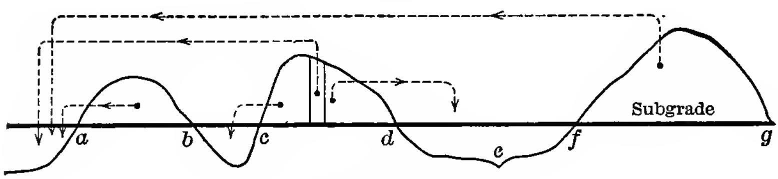
For the special case in which each of the swell-factors is unity, $s = 1$, and eqs. 29 and 30 reduce to

$$H_{28} - H_{29} = 2\,bcV. \qquad (31)$$



SHOWING NO CROSSHAUL

FIG. 32.



SHOWING CROSSHAUL

FIG. 33.

Looking at eqs. 30 and 31, we see that when the two swell-factors are equal, whether they are each equal to unity or not, crosshauling increases, never diminishes, the total haul.

From one point of view all crosshauling with the exception noted under eq. 29 is unnecessary hauling and therefore a waste of effort, time, and money; but often the conditions of the work are such that some crosshauling gives an operating advantage which more than offsets the extra cost due to the crosshauling. This is peculiarly the case in steam-shovel work.

It is quite possible to underestimate the extra cost due to crosshauling. A case in point which has come to the author's notice is illustrated in Figs. 32 and 33. As indicated by the arrows, there was a surplus of material on the right and a lack of material on the left. The arrows in Fig. 32 show a distribution involving the least haul, for there is no crosshaul. In moving the shovel from cut *cd* to cut *fg* a streamlet, *e*, and moderately soft bottom lands had to be crossed. To minimize the cost of making this move the contractor hauled material from cut *cd* to the right — against the general movement of material — making the fill *df* full height and of top width ample for the passage of the shovel. After the work was completed the contractor requested that the resulting crosshaul be taken into account in computing the overhaul, on the ground that the character of the depression made it practically necessary to carry out the work as described. The saving in moving expense, due to taking the shovel across from *d* to *f* on top of the fill instead of on the natural surface of the ground, was at an outside figure two hundred dollars. To save this amount the actual overhaul was increased by crosshaul some three hundred and sixty thousand station-yards. It is safe to say that the contractor did not realize how much it was costing him to make the saving.

It is the author's opinion that crosshaul should be excluded when computing overhaul except where crosshauling has been ordered by the engineer or where the contract or a special agreement directs otherwise.

44. **Effect of crosshaul within the free-haul limits, on overhaul.** — In Figs. 34 and 35 which show the same profile there is lack of material on the left and a surplus on the right; and this results in a general movement of material from right to left.

Fig. 35 shows free-haul material marked out by free-haul limits *p* and *r*, and by free-haul limits *v* and *x*, on the forward ends of the cuts.

Fig. 34 shows additional free-haul material marked out by free-haul limits *s* and *u*, on the supposition that between *s* and *u* the material is hauled backward — against the general movement.

Let C_c designate the body of cut *st*; C_a, of cut *yz*; F_b, of fill *tu*; and F_d, of fill *no*. The distribution in the two figures is the same except for this:

In Fig. 35,	cut C_c makes fill F_d;
and	cut C_a makes fill F_b;
while in Fig. 34,	cut C_c makes fill F_b;
and	cut C_a makes fill F_d.

There is no crosshaul in Fig. 35. In Fig. 34 there is crosshaul between s and u. The effect of this crosshaul on the total haul is shown in the preceding section. Now what is the effect of crosshaul within the free-haul limits s and u, on the overhaul?

Let a be the center of gravity of cut C_a; b, of fill F_b; c, of cut C_c; and d, of fill F_d.

Fills F_b and F_d have the same volume — V, say.

Let C_c = volume of cut C_c;
C_a = volume of cut C_a;
s_c = swell-factor of cut C_c;
s_a = swell-factor of cut C_a;
f = free-haul distance;
O_{34} = total overhaul on volumes C_c and C_a in Fig. 34;
O_{35} = total overhaul on volumes C_c and C_a in Fig. 35.

Let all distances be expressed in stations of 100 ft.

Then $$C_a = \frac{V}{s_a} \quad \text{and} \quad C_c = \frac{V}{s_c}.$$

In Fig. 34 the overhaul on C_c is nothing, because the material is all dumped within the free-haul distance. The overhaul on C_a is $C_a(ad - f)$. Hence the total overhaul on C_c and C_a in Fig. 34 is

$$\begin{aligned} O_{34} &= C_a(ad - f) \\ &= V\left(\frac{ab}{s_a} + \frac{bc}{s_a} + \frac{cd}{s_a} - \frac{f}{s_a}\right), \end{aligned} \tag{32}$$

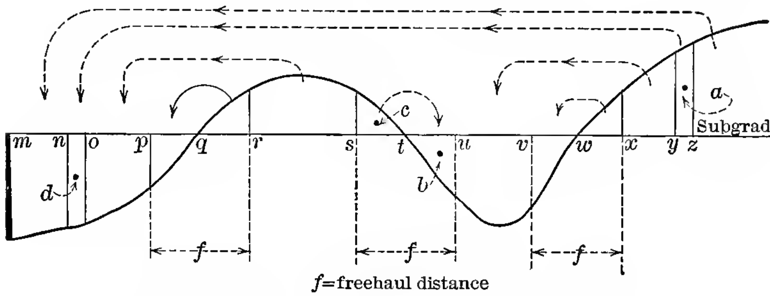
since $$C_a = \frac{V}{s_a} \text{ and } ad = ab + bc + cd.$$

In Fig. 35 the overhaul on C_c is

$$C_c(cd - f) = V\left(\frac{cd}{s_c} - \frac{f}{s_c}\right) \tag{33}$$

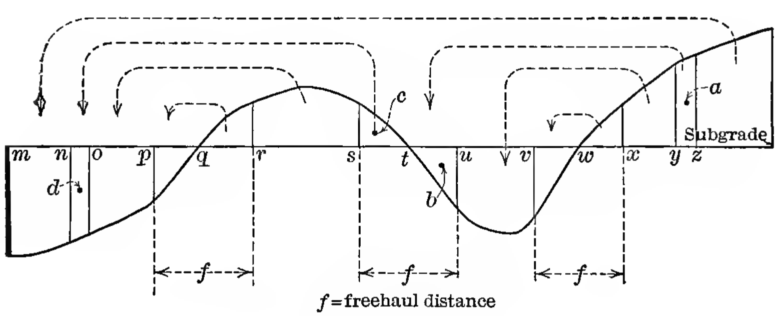
and the overhaul on C_a is

$$C_a(ab - f) = V\left(\frac{ab}{s_a} - \frac{f}{s_a}\right). \tag{34}$$



SHOWING CROSSHAUL WITHIN FREEHAUL LIMITS

FIG. 34.



SHOWING NO CROSSHAUL

FIG. 35.

Hence the total overhaul on C_a and C_c in Fig. 35 is

$$O_{35} = V\left(\frac{cd}{s_c} - \frac{f}{s_c} + \frac{ab}{s_a} - \frac{f}{s_a}\right). \tag{35}$$

The overhaul in Fig. 34, where the free haul material C_c is crosshauled, exceeds the overhaul in Fig. 35 by

$$O_{34} - O_{35} = V\left(\left(\frac{ab}{s_a} + \frac{bc}{s_a} + \frac{cd}{s_a} - \frac{f}{s_a}\right) - \left(\frac{cd}{s_c} - \frac{f}{s_c} + \frac{ab}{s_a} - \frac{f}{s_a}\right)\right)$$

$$= V\left(\frac{bc}{s_a} + cd\left(\frac{1}{s_a} - \frac{1}{s_c}\right) + \frac{f}{s_c}\right). \qquad (36)$$

$O_{34} - O_{35}$ will be negative when $\frac{bc}{s_a} + \frac{cd}{s_a} + \frac{f}{s_c} < \frac{cd}{s_c}$; and this is a possible condition.

When the swell-factors are the same for cuts C_c and C_a, $s_c = s_a$, $= s$, say, and eq. 36 reduces to

$$O_{34} - O_{35} = \frac{V}{s}(bc + f). \qquad (37)$$

When the swell-factors for cuts C_c and C_a are both unity, eq. 36 reduces to

$$O_{34} - O_{35} = V(bc + f). \qquad (38)$$

In view of the foregoing, it seems plain that when computing overhaul for material on a stretch of profile which indicates general movement of material in one direction, as shown in Figs. 34 and 35, free-haul limits should be established only on the forward end of each cut unless there is a previous agreement to the contrary. In other words, when it will not conflict with existing agreements, overhaul should be computed on the basis of Fig. 35 rather than on the basis of Fig. 34.

45. **Limit of error in total cost of overhaul resulting from error in computed center of gravity.** — In Fig. 1, g is the station and plus of the center of gravity of the cut mn as determined by some method (Chapter IV). Therefore g is in some degree approximate. Let us assume that we know that the true center of gravity of the cut mn lies within a distance E of g. Then the limit of uncertainty, or limit of error, in g is E. This may be

briefly expressed by saying that the ascertained station of the center of gravity of cut *mn* is $g \pm E$. Similarly, let us assume that the ascertained station and plus of the fill *op*, made by the cut *mn*, is $g' \pm E$.

Let V = volume of cut *mn*, in cubic yards;
$h = g' - g$ = average haul-distance (stas.) for volume V;
f = free-haul distance (stas.);
p = price of overhaul per station-yard;
P = total cost of overhaul on V;
E = limit of error in g and in g', E being expressed in stations;
E'' = limit of error in P due to errors E.

Then the total cost of overhaul on volume V is

$$P = V(h - f)p = Vhp - Vfp. \quad (39)$$

Now any error in P due to errors in g and g' must come through h. If the true center of gravity of cut *mn* lay a distance E to the right of g, and the true center of gravity of fill *op* lay a distance E to the left of g', the error in h would be $2E$. On the other hand, if the true center of gravity of cut *mn* lay at a distance E to the left of g, and the true center of gravity of fill *op* lay at a distance E to the right of g', the error in h would then be $-2E$. Thus the maximum possible error in h due to errors in g and g' is $\pm 2E$. The corresponding maximum possible error in P is

$$E'' = 2VEp. \quad (40)$$

Example. — Assume that by some method we have found sta. 18 + 65 (no figure) to be the center of gravity of a body of cut the volume of which is 1000 c.y.; and sta. 27 + 90 to be the center of gravity of the corresponding body of fill. Let us say that we know that the limit of error in the ascertained position of each of the two centers of gravity is 5 ft. Assume free-haul distance = 300 ft.; and the price of overhaul per station-yard = one cent. Then the cost of overhaul on the 1000 c.y. is, since $h = g' - g = 9.25$ stas., $P = 1000\ (6.25)\ (0.01) = \$62.50$. And the maximum possible error in this result due to errors in the ascertained positions of the two centers of gravity is by eq. 40, $E'' = 2\ (1000)\ (0.05)$

(0.01) = \$1. Observe that we do not say that the result, \$62.50, is in error to the amount of \$1. The fact is we cannot know what is the actual error in the result. We know only that the error due to the cause stated lies somewhere between + \$1 and − \$1. We do know, however, that in many cases in practice the error in the ascertained position of the two centers of gravity is as likely to be positive as negative; and hence in the long run these errors tend to neutralize one another.

46. Statement of overhaul. — After the overhaul has been computed, the next step is to make a statement of overhaul for the use of interested parties. This statement concerning the overhaul on any body of material should give not only the amount of the overhaul but also sufficient data to enable one to locate quickly and exactly on the profile the limits and center of gravity of that body of material both in cut and in fill, and to check readily the overhaul. It is believed that the form of statement given in Fig. 36 satisfies every requirement; and that no one of the columns of the form can be omitted without inconvenience.

47. Methods of overhaul computation. — The overhaul computations which appear in the following pages are all based on the overhaul clause recommended by the American Railway Engineering Association. See Sec. 38.

NOTE. — The overhaul clause is referred to in this book as the BASIS of overhaul computation rather than as the METHOD of computation, in order that the latter term may be free to be applied to each of the several ways — some arithmetical, some graphical, ranging in accuracy from the roughly approximate to the closely approximate — which are described and illustrated in the following pages.

Each of the following methods of computing overhaul on the American Railway Engineering Association basis, corresponds to one of the methods given in Chapter IV for determining the center of gravity. Each method of computing overhaul takes its number from the number of the method therein used to determine the center of gravity.

Methods of Computing Overhaul

48. **Method I.** — The center of gravity of each body of overhauled material in cut and in fill is determined by eye (Sec. 27) directly on the profile. The limits of any body of material overhauled and the limits of free haul are likewise determined by eye (Sec. 42). Method I is applied in Chapter VI (Secs. 57–66) to the profile of Fig. 19, and in Chapter VII (Secs. 148–156) to the profile of Fig. 38.

49. **Method II.** — The center of gravity of each body of overhauled material is assumed to lie at the center of length of that body. See Sec. 28. The limits of each body of material are determined by eye (Sec. 20 (c)) or by arithmetic (Sec. 20 (a)). The limits of free haul are determined by eye (Sec. 42) or by arithmetic (Sec. 40). This method of computing overhaul is given in Chapter VI (Secs. 67–77) by application to the profile of Fig. 19.

50. **Method III.** — The center of gravity of each body of overhauled material is determined by computing its distance from the center of length (Sec. 29). The limits of bodies of material are found by arithmetic (Sec. 20 (a)). The limits of free haul are determined by arithmetic (Sec. 40). The details of this method of overhaul computation are illustrated in Chapter VI (Secs. 78–88) by application to the profile of Fig. 19.

51. **Method IV.**—The center of gravity of each body of material overhauled is assumed to lie at its center of mass, and is found by arithmetic. See Sec. 30. The limits of bodies of material are found by arithmetic (Sec. 20 (a)). Limits of free haul are determined by arithmetic (Sec. 40). Method IV is illustrated in detail in Chapter VI (Secs. 89–99) by application to the profile of Fig. 19, and in Chapter VII (Secs. 157–167) to the profile of Fig. 38.

52. **Method V.** — The mass curve is used. The center of gravity of each body of material overhauled is assumed to lie at its center of mass. See Sec. 31. The limits of bodies of material are determined by the mass curve (Sec. 20 (b)). The

limits of free haul are determined by the mass curve (Sec. 41). This method of computing overhaul is applied in Chapter VI (Secs. 100–111) to the profile of Fig. 19, and in Chapter VII (Secs. 168–179) to the profile of Fig. 38.

A full description of this method, based on Sec. 31 (a), is given in Molitor and Beard's "Manual for Resident Engineers," pp. 57–61, and reprinted in Proceedings, American Railway Engineering Association, vol. 7 (1906), pp. 401–403. The same method, based on Sec. 31 (b), is given in *Engineering News*, March 14, 1891, pp. 254, 255 (reprinted in Proceedings, American Railway Engineering Association, vol. 7 (1906), pp. 386–389), in a contribution by Mr. T. S. Russell who gives credit for the method to Mr. R. P. Bruer.

53. **Method VI.** — Arithmetical computation. The center of gravity of each prismoid is assumed to lie at the middle of its length. The center of gravity of each body of overhauled material is determined by the method of moments (Sec. 32). The limits of bodies of material are determined by arithmetic (Sec. 20 (a)). The limits of free haul are found by arithmetic (Sec. 40). This method of computing overhaul is applied in Chapter VI (Secs. 112–122) to the profile of Fig. 19, and in Chapter VII (Secs. 180–190) to the profile of Fig. 38.

54. **Method VII.** — The mass curve is used. The center of gravity of each prismoid is assumed to lie at its center of length. The center of gravity of each body of material overhauled is found by the method of moments applied graphically by means of the mass curve. See Sec. 34. The limits of bodies of material are determined by the mass curve (Sec. 20 (b)). The limits of free haul are found by the mass curve (Sec. 41). Method VII is given in detail in Chapter VI (Secs. 123–134) through its application to the profile of Fig. 19, and in Chapter VII (Secs. 191–202) to the profile of Fig. 38.

55. **Method VIII.** — Arithmetical computation. The center of gravity of each prismoid is found by computing its distance

from the center of length of the volume. The center of gravity of each body of overhauled material is determined by the method of moments (Sec. 35). The limits of bodies of material are found by arithmetic (Sec. 20 (a)). The limits of free haul are determined by arithmetic (Sec. 40). This method of computing overhaul is applied in Chapter VI (Secs. 135–145) to the profile of Fig. 19.

CHAPTER VI

OVERHAUL COMPUTED FOR THE SIMPLE CASE OF FIG. 19

In this chapter the overhaul of Fig. 19 is computed by each of the eight methods of computing overhaul; the work under each method being laid out in formal steps, and in detail. The results are presented in Fig. 36 on the lines which begin with "A," and compared in Sec. 146. This chapter is intended to serve the computer who has chosen one of the eight methods of computing overhaul (Secs. 48–55), as a guide in his computations. The following chapter is similar to this, but the problem there solved is complex. (Fig. 19 faces p. 96; Fig. 36, p. 67.)

56. Preliminary remarks. — Each of the eight methods of computing overhaul, stated in the preceding chapter, is applied in turn to the simple case presented by the profile of Fig. 19.

The first step in the computation of overhaul, by whatever method of computation, is to gather all the data bearing on the problem in hand. The data for the overhaul problem of Fig. 19 are given in Step 1 of each method following.

A comparison of the results obtained by the several methods applied to the particular problem of Fig. 19, is made in Sec. 146.

Overhaul of Fig. 19 Computed by Method I

(In Method I center of gravity and all limits are determined by eye.)

57. Step 1. Data. — We are given the following information:

(1) We are given the profile, Fig. 19.

(2) The volume of each station and substation as computed from the notes of final cross-sections is given in Fig. 19.

(3) The material between 9 + 00 and the grade point (13 + 75) was used to make fill of full section from the grade point toward the right as far as the material would go.

(4) The swell-factor for the material of the cut is given as 1.25.

(5) The free-haul distance is given as 300 ft.

58. Step 2. Distribution of material. — See Sec. 57 (3).

59. Step 3. Swell-factor and equating-factor. — The swell-factor is 1.25 (Sec. 57 (4)). The equating-factor for the fill is therefore $\frac{1}{1.25}$ (Sec. 7) = 0.8.

60. Step 4. Limits of bodies of material. — The limits of the cut are 9 + 00 and 13 + 75 (Sec. 57 (3)). The right-hand limit of the fill is determined by eye, directly on the profile, by shifting the edge of a card (or of a piece of drawing paper, or of a triangle) held parallel to the vertical rulings of the profile paper, from the grade point toward the right until the volume of fill exposed to view appears to balance the cut between 9 + 00 and the grade point. The station and plus of the right-hand limit of the fill thus determined is 18 + 20. A mark is made on the profile at this point.

NOTE. — It must be borne in mind that the volumes are not proportional to the center heights or the profile areas.

61. Step 5. Limits of free haul. — These are determined by eye — the method of Sec. 42. They appear to be at 12 + 50 and 15 + 50. (Of course the pluses are made the same to correspond to the known free-haul distance.) A vertical line is penciled at each of these points.

62. Step 6. Centers of gravity. — Center of gravity is determined by eye — the method of Sec. 27. The center of gravity of the cut limited by 9 + 00 and 12 + 50 appears to be at 10 + 60; and a mark is made on the profile at this point. The center of gravity of the body of fill limited by 15 + 50 and 18 + 20 appears to be at 17 + 10; and this point is marked on the profile.

63. Step 7. Average haul-distance. — The average haul-distance for a given body of material is the distance between the center of gravity of that body in cut and the center of gravity of the same body in fill.

Center of gravity of fill (Sec. 62) = sta. 17 + 10.
Center of gravity of cut (Sec. 62) = sta. 10 + 60.

Average haul-distance = 6.50 stas.

64. Step 8. Average overhaul-distance. — This is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 63) = 6.50 stas.
Free-haul distance (Sec. 57 (5)) = 3.00 stas.

Overhaul distance = 3.50 stas.

65. Step 9. The overhaul. — The overhaul resulting from moving the material of cut, 9 + 00 to 12 + 50, to fill, 15 + 50 to 18 + 20, is equal to the volume of the said cut multiplied by the overhaul distance. The volume of the cut, 9 + 00 to 12 + 50, is about 1100 c.y. (Fig. 19). The overhaul distance for this volume is 3.50 stas. (Sec. 64). The overhaul = 1100 × 3.50 = 3850 sta-yds.

66. Step 10. Statement of overhaul. — When overhaul is found by Method I, it may not be worth while to make as complete a statement of it as that shown in Fig. 36. Nevertheless some orderly record of the work should be kept. If a series of computations is worth making, it seems worth the trouble to make a digest of the results. In order to permit a convenient comparison of the result of each Step by the present method, with the results of the corresponding Steps of the other methods of computing the overhaul of Fig. 19, a complete statement of the overhaul just computed is entered in Fig. 36 on line A–I.

Overhaul of Fig. 19 Computed by Method II

(In Method II center of gravity of body of cut or fill is assumed to lie at center of length. Limits are computed by arithmetic.)

67. Step 1. Data. — We are given the following information:

(1) We are given the profile, Fig. 19.

(2) The volume of each station and substation as computed from the notes of final cross-sections is given in Fig. 19.

(3) The material between 9 + 00 and the grade point (13 + 75) was used to make fill of full section from the grade point toward the right as far as the material would go.

(4) The swell-factor for the material of the cut is given as 1.25.

(5) The free-haul distance is given as 300 ft.

68. Step 2. Distribution of material. — See Sec. 67 (3).

69. Step 3. Swell- and equating-factors. — The swell-factor is 1.25. The equating-factor is therefore $\frac{1}{1.25} = 0.8$ (Sec. 7).

70. Step 4. Equate each station-volume of fill to volume in place. — Each station-volume of fill is multiplied by the equating-factor (Sec. 69). The resulting volumes in place are entered on Fig. 19.

71. Step 5. Limits of bodies of material. — The limits of the cut are 9 + 00 and 13 + 75 (Sec. 67 (3)). The left-hand limit of the fill is at the grade point 13 + 75. The right-hand limit of the fill made from the material of the cut is found by the arithmetical method of Sec. 20 where for this particular case the right-hand limit of the fill is computed to be 18 + 25.

72. Step 6. Limits of free haul. — The free-haul limits are determined by the method of Sec. 40 where for this profile the limits were computed to be 12 + 28 and 15 + 28. See Sec. 40 for the details of the computation.

73. Step 7. Centers of gravity. — Centers of gravity are determined by Method II, Sec. 28, in which it is assumed that the center of gravity of a body of material lies at the mid-point of that body. The center of gravity of the cut, 9 + 00 to 12 + 28, was found in Sec. 28 to lie at 10 + 64. The center of gravity of the fill, 15 + 28 to 18 + 25, is found by this method to lie at (sta. 15 + 28) + $\frac{1}{2}$ [(sta. 18 + 25) − (sta. 15 + 28)] = (sta. 15 + 28) + 148.5 = sta. 16 + 76.5.

74. **Step 8. Average haul-distance.** — The average haul-distance for the material between 9 + 00 and 12 + 28 is the distance between the center of gravity of cut, 9 + 00 to 12 + 28, and the center of gravity of the fill, 15 + 28 to 18 + 25.

Center of gravity of fill (Sec. 73) = sta. 16 + 76.5.
Center of gravity of cut (Sec. 73) = sta. 10 + 64.

Average haul-distance = 6.125 stas.

75. **Step 9. Average overhaul-distance.** — This is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 74) = 6.125 stas.
Free-haul distance (Sec. 67 (5)) = 3. stas.

Overhaul distance = 3.125 stas.

76. **Step 10. The overhaul.** — The overhaul on the material of cut, 9 + 00 to 12 + 28, is equal to the volume of that cut multiplied by the overhaul distance. The volume of the cut is 1056 c.y. (Fig. 19). The overhaul distance is 3.125 stas. (Sec. 75). Therefore the overhaul = 1056 × 3.125 = 3300 sta-yds.

77. **Step 11. Statement of overhaul.** — A full statement of the overhaul just computed is entered on line A–II, Fig. 36.

Overhaul of Fig. 19 Computed by Method III

(In Method III center of gravity of body of cut or fill is determined by computing its distance from center of length. Limits are computed by arithmetic.)

78. **Step 1. Data.** — We are given the following information:

(1) We are given the profile, Fig. 19.

(2) The volume of each station and substation as computed from the notes of final cross-sections is given on Fig. 19.

(3) The material between 9 + 00 and the grade point (13 + 75) was used to make fill of full section from the grade point toward the right as far as the material would go.

(4) The swell-factor for the material of the cut is given as 1.25.
(5) The free-haul distance is given as 300 ft.

79. Step 2. Distribution of material. — See Sec. 78 (3).

80. Step 3. Swell- and equating-factors. — The swell-factor is 1.25 (Sec. 78 (4)). The equating-factor is therefore $\frac{1}{1.25} = 0.8$ (Sec. 7).

81. Step 4. Equate each station-volume of fill to volume in place. — Each station-volume of the fill is multiplied by the equating-factor 0.8 (Sec. 80). The resulting volumes in place are entered in parentheses on Fig. 19.

82. Step 5. Limits of bodies of material. — The limits of the cut are 9 + 00 and 13 + 75 (Sec. 78 (3)). The left-hand limit of the fill is at the grade point 13 + 75. The right-hand limit of the fill made from the material of the cut is found by the arithmetical method of Sec. 20, where for this case the right-hand limit of the fill is computed to be at 18 + 25.

83. Step 6. Limits of free haul. — The free-haul limits are determined by the method of Sec. 40, where for this case the limits were computed to be 12 + 28 and 15 + 28.

84. Step 7. Centers of gravity. — Use Method III, Sec. 29. In Sec. 29 the center of gravity of cut, 9 + 00 to 12 + 28, was computed to lie at 10 + 36.7. (See Sec. 29 for the details of the computation.) The computation for the center of gravity of fill, 15 + 28 to 18 + 25, is made thus: The mid-point is at (sta. 15 + 28) + $\frac{1}{2}$ [(sta. 18 + 25) − (sta. 15 + 28)] = 16 + 76.5. Area of section at 15 + 28 is 102 sq. ft. The area of the section at 18 + 25 is 180 sq. ft. (Fig. 19). The length of the body of fill is (sta. 18 + 25) − (sta. 15 + 28) = 297 ft. Compute $x = \frac{297}{6}\left(\frac{180 - 102}{180 + 102}\right) = 13.7$ ft. The center of gravity sought lies x feet to the right of the mid-point, that is, at (sta. 16 + 76.5) + 13.7 = sta. 16 + 90.2.

85. Step 8. Average haul-distance. — The average haul-distance for the material between 9 + 00 and 12 + 28 is the distance between the center of gravity of cut, 9 + 00 to 12 + 28, and the center of gravity of the fill, 15 + 28 to 18 + 25.

Center of gravity of fill (Sec. 84)	=	sta. 16 + 90.2
Center of gravity of cut (Sec. 84)	=	sta. 10 + 36.7
Average haul-distance	=	6.535 stas.

86. Step 9. Average overhaul-distance. — The average overhaul-distance is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 85)	=	6.535 stas.
Free-haul distance (Sec. 78 (5))	=	3.000 stas.
Average overhaul-distance	=	3.535 stas.

87. Step 10. The overhaul. — The overhaul on the material of cut, 9 + 00 to 12 + 28, is equal to the volume of that material multiplied by its overhaul distance. The volume of the cut is 1056 c.y. (Fig. 19). The average overhaul-distance is 3.535 stas. (Sec. 86). Thus the overhaul = 1056 × 3.535 = 3730 sta-yds.

88. Step 11. Statement of overhaul. — The complete statement of the overhaul just computed is entered in Fig. 36 on line A–III.

Overhaul of Fig. 19 Computed by Method IV

(In Method IV center of gravity of body of cut or fill is assumed to lie at its center of volume, and is computed by arithmetic. Limits are computed by arithmetic.)

89. Step 1. Data. — We are given the following information:

(1) We are given the profile, Fig. 19.

(2) The volume of each station and substation as computed from the notes of final cross-sections is given on Fig. 19.

(3) The material between 9 + 00 and the grade point (13 + 75) was used to make fill of full section from the grade point toward the right as far as the material would go.

(4) The swell-factor for the material of the cut is given as 1.25.
(5) The free-haul distance is given as 300 ft.

90. Step 2. Distribution of material. — See Sec. 89 (3).

91. Step 3. Swell- and equating-factors. — The swell-factor is 1.25 (Sec. 89 (4)). The equating-factor is therefore $\frac{1}{1.25} = 0.8$ (Sec. 7).

92. Step 4. Equate each station-volume of fill to volume in place. — Each station-volume of fill is multiplied by the equating-factor 0.8 (Sec. 91). The resulting volumes in place are entered in Fig. 19.

93. Step 5. Limits of bodies of material. — The limits of the cut are 9 + 00 and 13 + 75 (Sec. 89 (3)). The left-hand limit of the fill is at the grade point 13 + 75. The right-hand limit of the fill made from the material of the cut is found by the arithmetical method of Sec. 20, where for this case the right-hand limit of the fill is computed to be at 18 + 25.

94. Step 6. Limits of free haul. — The free-haul limits are determined by the method of Sec. 40, where for this case the limits of free haul were computed to be 12 + 28 and 15 + 28.

95. Step 7. Centers of gravity. — Use Method IV, Sec. 30, where it is assumed that the center of gravity lies at the center of mass. In Sec. 30 it was computed that the center of gravity of cut, 9 + 00 to 12 + 28, lies at 10 + 09.3. By the same method the center of gravity of fill, 15 + 28 to 18 + 25, is found to lie at 17 + 03.

96. Step 8. Average haul-distance. — The average haul-distance for the material of cut, 9 + 00 to 12 + 28, is the distance between the center of gravity of that cut and the center of gravity of fill, 15 + 28 to 18 + 25.

Center of gravity of fill (Sec. 95) = sta. 17 + 03.
Center of gravity of cut (Sec. 95) = sta. 10 + 09.3.

Average haul-distance = 6.937 stas.

97. **Step 9. Average overhaul-distance.** — This is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 96)	=	6.937 stas.
Free-haul distance (Sec. 89 (5))	=	3.000 stas.
Average overhaul-distance	=	3.937 stas.

98. **Step 10. The overhaul.** — The overhaul on the material of cut, 9 + 00 to 12 + 28, is equal to the volume of that material multiplied by its average overhaul-distance. The volume of the material is 1056 c.y. (Fig. 19). The average overhaul-distance is 3.937 stas. (Sec. 97). Therefore the overhaul = 1056 × 3.937 = 4157 sta-yds.

99. **Step 11. Statement of overhaul.** — A full statement of the overhaul just computed is entered on line A–IV, Fig. 36.

Overhaul of Fig. 19 Computed by Method V

(In Method V center of gravity of body of cut or fill is assumed to lie at its center of volume, and is determined by mass curve. Limits are determined by mass curve.)

100. **Step 1. Data.** — We are given the following information:

(1) We are given the profile, Fig. 19.

(2) The volume of each station and substation as computed from the notes of final cross-sections is given on Fig. 19.

(3) The material between 9 + 00 and the grade point (13 + 75) was used to make fill of full section from the grade point toward the right as far as the material would go.

(4) The swell-factor for the material of the cut is given as 1.25.

(5) The free-haul distance is given as 300 ft.

101. **Step 2. Distribution of material.** — See Sec. 100 (3).

102. **Step 3. Swell- and equating-factors.** — The swell-factor is 1.25 (Sec. 100 (4)). The equating-factor is therefore $\frac{1}{1.25} = 0.8$ (Sec. 7).

103. Step 4. Equate each station-volume of fill to volume in place. — Multiply each station-volume of the fill by the equating-factor 0.8 (Sec. 102). The resulting volumes in place are entered in Fig. 19.

104. Step 5. Plot the mass curve. — The mass curve is plotted from cut-volumes and equated fill-volumes, in the manner explained in Sec. 10. In Fig. 19 sta. 9 + 00 is taken as the origin for the mass curve.

105. Step 6. Limits of bodies of material. — The left-hand limit of the cut is given as 9 + 00. We find the right-hand limit of the fill by the mass-curve method of Sec. 20. Through *A* — 9 + 00 on the mass curve — draw a horizontal to meet the right slope of the curve at some point *B*. In this particular case the horizontal, thus drawn, happens to coincide with the base line. *B* is the right-hand limit of the fill made from the material of the cut, 9 + 00 to 13 + 75. By means of the engineer's scale we find that *B* is at 18 + 24. (If the plus of the point *B* had been estimated by eye it would probably have been taken as 25 ft. instead of 24 ft.) The plus, 24 ft., is now recorded in proper position on the mass curve in Fig. 19.

106. Step 7. Limits of free haul. — The free-haul distance is given as 300 feet (Sec. 100 (5)). By the method of Sec. 41, we find that a horizontal line 300 ft. long which has its extremities *C* and *D* on the curve must have one extremity at 12 + 29 and the other at 15 + 29. These two stations are the free-haul limits. (The plus of each limit was scaled in this case. The position of *C* was found to be 12 + 30, and that of *d*, 15 + 28. As the two limits must be just 300 ft. apart, a compromise was made on the plus of 29 ft.)

107. Step 8. Centers of gravity. — The centers of gravity are determined by Method V, Sec. 31, thus: Mark the point C'' midway between the free-haul line *CD* and the base line *AB*. Draw through C'' a horizontal to meet the mass curve at the two points *M* and M'. *M* is the center of gravity of cut, 9 + 00 to 12 + 29, and, by scale, lies at 10 + 10. M' is the center of gravity of the fill, 15 + 29 to 18 + 24, and lies, by scale, at 17 + 05.

108. **Step 9. Average haul-distance.** — The average haul-distance for the material of cut, 9 + 00 to 12 + 29, is the distance between the center of gravity of that material and the center of gravity of the fill, 15 + 29 to 18 + 24.

Center of gravity of fill (Sec. 107) = sta. 17 + 05.
Center of gravity of cut (Sec. 107) = sta. 10 + 10.

Average haul-distance = 6.95 stas.

109. **Step 10. Average overhaul-distance.** — This is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 108) = 6.95 stas.
Free-haul distance (Sec. 100 (5)) = 3.00 stas.

Average overhaul-distance = 3.95 stas.

110. **Step 11. The overhaul.** — The overhaul on the material of cut, 9 + 00 to 12 + 29, is equal to the volume of that material multiplied by its average overhaul-distance. The volume of the cut was found by scaling the ordinate CC' to be 1064 c.y. (Of course in this particular case it would have been easy to find the exact volume, 1056 c.y., which was originally used in plotting the ordinate CC'. See footnote to Sec. 34, p. 47.) The average overhaul-distance is 3.95 (Sec. 109). Therefore the overhaul is 1064 × 3.95 = 4202 station-yards.

111. **Step 12. Statement of overhaul.** — The complete statement of the overhaul computed above is entered in Fig. 36 on line A–V.

Overhaul of Fig. 19 Computed by Method VI

(In Method VI the center of gravity of a body of cut or fill is computed arithmetically by the method of moments; the center of gravity of each station-volume being assumed to lie at its center of length. The limits are computed by arithmetic.)

Station.	g	V	a	Va	b	Vb
15+28						
16	15+64	216	.36	78	2.61	564
17	16+50	300	1.22	366	1.75	525
18	17+50	400	2.22	888	.75	300
18+25	18+12.5	140	2.845	398	.125	17
Totals..	....	1056	....	1730	....	1406

g = center of gravity of station-volume on the assumption that center of gravity is at the mid-point.

V = equated station-volume in cubic yards. (The result of this computation would be the same if we used the actual station-volumes, as pointed out in Sec. 33.)

a = length of lever-arm (stas.) for the pivot 15 + 28 = distance from g to sta. 15 + 28.

Va = moment (sta-yds.) of V about sta. 15 + 28.

b = lever-arm (stas.) for the pivot 18 + 25 = distance from g to sta. 18 + 25.

Vb = moment (sta-yds.) of V about sta. 18 + 25.

Average lever-arm about sta. 15 + 28 = distance from center of gravity of fill to sta. 15 + 28 $\Big\} = \frac{1730}{1056} = 1.638$ stas.

Hence the center of gravity of the fill is at (sta. 15 + 28) + 1.638 stas. = sta. 16 + 91.8.

To check this result we find the

Average lever-arm about sta. 18 + 25 = distance from center of gravity of fill to sta. 18 + 25 $\Big\} = \frac{1406}{1056} = 1.332$ stas.

Hence the center of gravity of the fill is at (sta. 18 + 25) − 1.332 stas. = sta. 16 + 91.8, which agrees with the result obtained above.

119. Step 8. Average haul-distance. — The average haul-distance for the material between stas. 9 and 12 + 28 is the distance between the two centers of gravity which we found in the preceding section:

Center of gravity of cut (Sec. 118) = sta. 10 + 30.3.
Center of gravity of fill (Sec. 118) = sta. 16 + 91.8.

Average haul-distance = 6.615 stas.

120. Step 9. Average overhaul-distance. — The average overhaul-distance is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 119) = 6.615 stas.
Free-haul distance (Sec. 112) = 3.000 stas.

Average overhaul-distance = 3.615 stas.

121. Step 10. The overhaul. — The overhaul is equal to the product of the volume of the cut between stas. 9 and 12 + 28 by the average overhaul-distance. The overhaul distance having been found in the preceding section to be 3.615 stas., and the volume of the cut, in Sec. 118, to be 1056 c.y., the overhaul is 1056 × 3.615 = 3817 sta-yds.

NOTE. — It should be noted here that the overhaul which we have found above might have been obtained in Sec. 118 simply by the addition of the sums of two columns of moments. Imagine (1) that all the prismoids of the cut were moved to the free-haul limit, sta. 12 + 28, and there united to form a single body; (2) that the single body was moved to the free-haul limit, sta. 15 + 28, and there formed into prismoids to suit the requirements of the fill; and (3) that each newly formed prismoid was then moved to its destined position in the fill. Evidently, the haul performed in (2) is free haul, while that performed in (1) and (3) is overhaul. Now, the haul under (1) is equal to the sum of the moments of the cut-volumes about sta. 12 + 28, or 2088 sta-yds. (Sec. 32); and the haul under (3) is equal to the sum of the moments of the (equated) fill-volumes about sta. 15 + 28, or 1730 sta-yds. (Sec. 118); and, therefore, the total overhaul is 2088 + 1730 = 3818 sta-yds. (checking the overhaul found above, in this section, within 1 sta-yd.). This short-cut to overhaul is advantageous only when centers of gravity and average haul-distance are not wanted. No formal statement of overhaul is satisfactory, however, unless it gives centers of gravity and average haul-distances.

122. Step 11. **Statement of overhaul.** — The statement of the overhaul just computed is given on line A–VI of Fig. 36.

Overhaul of Fig. 19 Computed by Method VII

(In Method VII the center of gravity of each body of cut or fill is computed by means of the mass curve by the method of moments. All limits are determined by the mass curve.)

123. Step 1. **Data.** — We are given the following information:

(1) We are given the profile, Fig. 19.

(2) The volume of each station and substation as computed from the notes of final cross-sections is given on Fig. 19.

(3) The material between 9 + 00 and the grade point (13 + 75) was used to make fill of full section from the grade point toward the right as far as the material would go.

(4) The swell-factor for the material of the cut is given as 1.25.

(5) The free-haul distance is given as 300 ft.

124. Step 2. **Distribution of material.** — See Sec. 123 (3).

125. Step 3. **Swell- and equating-factors.** — The swell-factor is 1.25 (Sec. 123 (4)). Therefore the equating-factor is $\frac{1}{1.25} = 0.8$ (Sec. 7).

126. Step 4. **Equate each station-volume of fill to volume in place.** — Multiply each station-volume of the fill by the equating-factor 0.8 (Sec. 125). The resulting volumes in place are entered in Fig. 19.

127. Step 5. **Plot the mass curve.** — The mass curve is plotted from cut-volumes and equated fill-volumes in the manner explained in Sec. 10. In Fig. 19 sta. 9 + 00 is taken as the origin of the mass curve.

128. Step 6. **Limits of bodies of material.** — The left-hand limit of the cut is given as 9 + 00. We find the right-hand limit of the fill by the mass-curve method of Sec. 20. Through *A* — 9 + 00 on the mass curve — draw a horizontal to meet the right slope of the curve at some point *B*. The horizontal, thus drawn,

happens, in this case, to coincide with the base line. *B* is the right-hand limit of the fill made from the material of the cut, 9 + 00 to 13 + 75. By means of the engineer's scale we find that *B* is at 18 + 24. (If the plus of *B* had been estimated by eye, it would probably have been taken as 25 ft. instead of 24 ft.) The plus, 24 ft., is now recorded in proper position on the mass curve in Fig. 19.

129. Step 7. Limits of free haul. — The free-haul distance is given as 300 ft. By the mass-curve method of Sec. 41 we find that a horizontal line 300 ft. long which has its extremities, *C* and *D*, on the curve must have one extremity at sta. 12 + 29 and the other at sta. 15 + 29. These two stations are the free-haul limits. (The plus of each limit was scaled in this case. The position of *C* was found to be 12 + 30 and that of *D*, 15 + 28. As the two limits must be just 300 ft. apart, a compromise was made on the plus of 29 ft.)

130. Step 8. Centers of gravity. — The centers of gravity are found by Method VII, Sec. 34, thus: Draw a vertical through *C* cutting *AB* at *C′*. We measure the area *ACC′A* with a planimeter and find it to be 2.1312 sq. in. (This area may be computed from the dimensions of the component trapezoids.) Since one square inch of paper represents 2 × 500 = 1000 sta-yds., the area *ACC′A* represents 2.1312 × 1000 = 2131 sta-yds. of haul. The ordinate *CC′* is found, by scaling, to represent 1064 c.y. (See Sec. 110.) Thus the distance from the center of gravity, *E*, of the cut *AC* to the right-hand limit, *C*, of the cut is

$$EE' = \frac{\text{area } ACC'A \text{ (in station-yards)}}{\text{ordinate } CC' \text{ (in cubic yards)}}$$

$$= \tfrac{2131}{1064} = 2.003 \text{ stas.}$$

E′, like *C*, is at	sta. 12 + 29
EE′ =	2 00.3
Therefore *E*, the center of gravity of cut, sta. 9 to sta. 12 + 29, is at	sta. 10 + 28.7

Similarly we find that the distance between the left-hand limit, D, of fill DB and the center of gravity, F, of that fill, is

$$FF' = \frac{\text{area } BDD'B \text{ (in station-yards)}}{\text{ordinate } DD' \text{ (in cubic yards)}}$$

$$= \tfrac{1776}{1064} = 1.67 \text{ stas.}$$

F', like D, is at	sta. 15 + 29
FF' =	1 67
Therefore F, the center of gravity of fill DB, is at	sta. 16 + 96.

131. Step 9. Average haul-distance. — The average haul-distance for the material of the cut, sta. 9 + 00 to sta. 12 + 29, is the distance between E, the center of gravity of this cut, and F, the center of gravity of the corresponding fill, stas. 15 + 29 to 18 + 24. From Sec. 130 we have

E at	sta. 10 + 28.7,
F at	sta. 16 + 96.0.
Average haul-distance = EF	= 6.673 stas.

132. Step 10. Average overhaul-distance. — The average overhaul-distance for the material in the cut between stas. 9 and 12 + 29 is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 131)	= 6.673 stas.
Free-haul distance (Sec. 123 (5))	= 3.00 stas.
Average overhaul-distance	= 3.673 stas.

The average overhaul-distance is evidently equal to EE' plus FF', also. See Fig. 19.

133. Step 11. The overhaul. — The overhaul on the material moved from cut AC to fill DB is equal to the volume of cut AC multiplied by the average overhaul-distance. Volume of cut is 1064 c.y. (Sec. 130) and the average overhaul-distance is 3.673 stas. (Sec. 132). Therefore the overhaul = 1064 × 3.673 = 3907 sta-yds.

NOTE 1. — The sum of the areas $ACC'A$ and $BDD'B$, expressed in station-yards, is the overhaul in this case (Sec. 14). In Step 8, Sec. 130, we might have found the overhaul directly, thus:

Area $ACC'A$	represents	2131	sta-yds.	of haul,
" $BDD'B$	"	1776	"	"
The total is		3907	"	"

and all of this is overhaul.

NOTE 2. — For Steps 8, 9, 10, 11, we might substitute the following procedure: (1) Find the whole area $ABDCA$ in square inches, by means of the planimeter or otherwise, and convert into station-yards of haul, thus obtaining the total haul on the material in cut AC. (2) Divide this total haul by the volume in cubic yards represented by the vertical distance between AB and CD, thus obtaining the average haul-distance. (3) Subtract the free-haul distance from this average haul-distance, obtaining the average overhaul-distance. (4) Multiply the average overhaul-distance by the volume of cut AC and thus obtain the overhaul required.

It would still remain to find the positions of the centers of gravity if these are desired for the statement of overhaul.

134. Step 12. Statement of overhaul. — The statement of the overhaul just computed is entered on line A–VII, Fig. 36.

Overhaul of Fig. 19 Computed by Method VIII

(In Method VIII the center of gravity of each cut or fill is found by the method of moments, using arithmetic; the center of gravity of each station-volume being found by computing its distance from the center of length. All limits are computed by arithmetic.)

135. Step 1. Data. — We are given the following information:

(1) We are given the profile, Fig. 19.

(2) The volume of each station and substation as computed from the notes of final cross-section is given on Fig. 19.

(3) The material between 9 + 00 and the grade point (13 + 75) was used to make fill of full section from the grade point toward the right as far as the material would go.

(4) The swell-factor for the material of the cut is given as 1.25.

(5) The free-haul distance is given as 300 ft.

136. Step 2. Distribution of material. — See Sec. 135 (3).

137. Step 3. Swell- and equating-factors. — The swell-factor is 1.25 (Sec. 135 (4)). The equating-factor is therefore $\frac{1}{1.25} = 0.8$ (Sec. 7).

138. Step 4. Equate each station-volume of fill to volume in place. — We multiply each station-volume of fill by the equating-factor 0.8 (Sec. 137). The equated fill-volumes have been entered on Fig. 19.

139. Step 5. Limits of bodies of material. — The limits of the cut are 9 + 00 and 13 + 75 (Sec. 135 (3)). The left-hand limit of the fill is 13 + 75. The right-hand limit of the fill made from the material of the cut is found by the arithmetical method of Sec. 20. It was found in Sec. 20 that the right-hand limit of the fill is 18 + 25.

140. Step 6. Limits of free haul. — The free-haul limits are determined by the method of Sec. 40. In Sec. 40 the free-haul limits for Fig. 19 were found to be stas. 12 + 28 and 15 + 28.

141. Step 7. Centers of gravity. — Use Method VIII, Sec. 35. By this method the center of gravity of the cut, 9+00 to 12+28, was found to lie at 10 + 27.3. Applying the same method to find the center of gravity of the fill, 15 + 28 to 18 + 25, we have the following computations:

COMPUTATION FOR CENTER OF GRAVITY OF EACH STATION-BODY

Station.	l	A	Diff. between end areas.	Sum of end areas.	$\frac{\text{Diff.}}{\text{Sum}}$.	$\frac{l}{6}$.	x	Center of sta. body.	Center of gravity of sta. body.
15+28		102							
	72		0	204	0.0	12.00	0.0	15+64	15+64
16		102							
	100		0	204	0.0	16.7	0.0	16+50	16+50
17		102							
	100		66	270	0.24	16.7	4.0	17+50	17+54
18		168							
	25		12	348	0.035	4.17	0.14	18+12.5	18+12.6
18+25		180							

l = length (ft.) of prismoid (station body);
A = end area (sq. ft.);
x = distance (ft.) from mid-point to center of gravity of prismoid.

COMPUTATION OF MOMENTS

Station.	g	V	a	Va	b	Vb
15+28						
	15+64	216	0.36	78	2.61	564
16						
	16+50	300	1.22	366	1.75	525
17						
	17+54	400	2.26	904	0.71	284
18						
	18+12.6	140	2.846	398	0.124	17
18+25						
	Totals...	1056	....	1746	....	1390

g = center of gravity of station body, as computed above.

V = equated station-volume in cubic yards. (The result of this computation would be the same if we used the actual fill-volumes instead of the equated (Sec. 33).)

a = length of lever-arm (stas.) for pivot at sta. 15 + 28 = distance from center of gravity of station body to sta. 15 + 28.

Va = moment (sta. yds.) of station-volume about sta. 15 + 28.

b = length of lever-arm (stas.) for pivot at sta. 18 + 25 = distance from center of gravity of station body to sta. 18 + 25.

Vb = moment (sta-yds.) of the station-volume about sta. 18 + 25.

Average lever-arm about sta. 15 + 28 = distance from center of gravity of fill to sta. 15 + 28 $= \frac{1746}{1056} = 1.653$ stas.

Hence the center of gravity of the fill is at (sta. 15 + 28) + 1.653 stas. = sta. 16 + 93.3.

To check this result we find the

Average lever-arm about sta. 18 + 25 = distance from center of gravity of fill to sta. 18 + 25 $= \frac{1390}{1056} = 1.316$ stas.

Hence the center of gravity of the fill is at (sta. 18 + 25) − 1.316 stas. = sta. 16 + 93.4 (check within 0.1 ft.)

142. Step 8. Average haul-distance. — The average haul-distance for the material of the cut, 9 + 00 to 12 + 28, is the distance between the center of gravity of that material and the center of gravity of the fill, 15 + 28 to 18 + 25.

Center of gravity of fill (Sec. 141) = sta. 16 + 93.3.
Center of gravity of cut (Sec. 141) = sta. 10 + 27.3.
Average haul-distance = 6.66 stas.

143. Step 9. Average overhaul-distance. — This is equal to the average haul-distance less the free-haul distance.

Average haul-distance (Sec. 142) = 6.66 stas.
Free-haul distance (Sec. 135 (5)) = 3.00 stas.

Average overhaul-distance = 3.66 stas.

144. Step 10. The overhaul. — The overhaul on the material of the cut, 9 + 00 to 12 + 28, is equal to the volume of that material multiplied by its average overhaul-distance. The volume of the material is 1056 c.y. (Fig. 19). The average overhaul-distance is 3.66 (Sec. 143). Therefore the overhaul = 1056 × 3.66 = 3865 sta-yds.

145. Step 11. Statement of overhaul. — The complete statement of the overhaul computed above is entered in Fig. 36, on line A–VIII.

Comparison of the Several Methods of Overhaul Computation

146. Comparison of results. — The overhaul of Fig. 19 has now been computed by each of the eight methods described in this book. For convenience the results, taken from column 18 of Fig. 36, are tabulated below. Since Method VIII is the most exact we shall assume that the overhaul computed by this method is the true overhaul, and compute the absolute and relative error of each result on this basis.

Method.	Overhaul (sta-yds.).	Error (sta-yds.).	Error, per cent.
I	3850	− 15	− 0.4
II	3300	−565	−14.6
III	3730	−135	− 3.5
IV	4157	+292	+ 7.6
V	4202	+337	+ 8.7
VI	3817	− 48	− 1.2
VII	3907	+ 42	+ 1.1
VIII	3865	0	0.0

The error in the overhaul computed by Method I is only 0.4%; but if several men, in turn, applied Method I to the profile, their results would vary widely. The 0.4% means only that this was a particularly lucky estimate. See Sec. 203 where this method is applied by two men to the overhaul of Fig. 38.

The case is different with Method II. Given the same data, any number of computers must, by this method, obtain the same result if they make no mistakes in the arithmetical operations. The error of − 14.6% in the result obtained by this method is a true measure of the accuracy of the method for the conditions of the profile. The error is, of course, due entirely to the fact that the body of material overhauled is not symmetrical about its center of length, either in cut or in fill. The left half of the cut is the heavier and the right half of the fill is the heavier, so that when we assume, as we do in Method II, that the center of gravity coincides with the center of length, we obtain an average haul-distance which is short of the true one; and this is shown by the minus sign of the error.

The error in the result obtained by Method III is − 3.5%; and this is a true measure of the accuracy of the method for the conditions of the profile; for the error is determined by the data and the assumption relative to center of gravity, and in no way is it dependent on the judgment of the computer.

The error, + 7.6%, in the result obtained by Method IV, is a true measure of the accuracy of this method under the conditions presented by the profile of Fig. 19; for the error is due entirely to the assumption that the center of gravity coincides with the center of volume, and not at all to the process of computing. Given the data, this method will give the same error in the result, whoever may be the computer.

Method V differs from Method IV in this way only: Method IV employs arithmetical, and Method V, mass-curve computation. Therefore while we should expect two or more computers to reach the same result by Method IV, we should expect them to obtain more or less discordant results by Method V. Any difference between the result obtained by Method V and that by Method IV is due to those errors which occur in plotting and scaling the mass curve. Let us see what is the maximum possible error in the result, as found by the mass curve. If the total possible error in plotting a distance is 0.01 inch, and the total possible error in scaling a distance is the same, the maximum resultant error in the yardage, as scaled from the mass curve, is $0.02 \times 500 = 10$ c.y. So the volume of the overhauled material, scaling 1064 c.y., may be as small as $1064 - 10 = 1054$, or as great as $1064 + 10 = 1074$; and the distance overhauled, scaling 3.95 stas., may be as little as $3.95 - (0.02 \times 4.00) = 3.87$ stas., or as much as $3.95 + 0.08 = 4.03$ stas. Therefore the true overhaul (true taking Method IV as the standard) *may* be as little as $1054 \times 3.87 = 4079$ sta-yds., or as much as $1074 \times 4.03 = 4328$ sta-yds. That the result, 4202 sta-yds., which has been obtained by Method V is so near the result from Method IV — so far within the extremes computed above — is due to the fact that of the several errors made in plotting and scaling, some have been negative and some positive, and the two kinds of error largely neutralize each other. It is because errors of graphical work are as likely to be positive as negative, and very large errors are rare, that the actual resultant error in the overhaul for one body of material computed by the mass curve is seldom so large as the computed limit of error; and that the error in the sum of several items of overhaul thus computed is generally insignificant. See Sec. 203.

Method VI gives a result in error by -1.2%; and this result, too, is determined by the assumptions relative to the position of the center of gravity of each station-volume upon which the method is based. The judgment of the computer has no effect on the result. The error obtained is a true measure of the accuracy of the method under the conditions of the given profile.

Method VII differs from Method VI only as Method V differs from Method IV; Methods VI and VII are based on the assumption that the center of gravity and center of length of a station volume are coincident, but the computations of the latter method are performed graphically (using the mass curve), and of the former, arithmetically. Hence the error in the result obtained by Method VII is not a true measure of the accuracy of the method.

In the foregoing, it has been assumed that the result, 3865 sta-yds. of overhaul obtained by the use of Method VIII, is without error because of the refined process of determining the center of gravity of a given body of material. We must remember, however, that the limits of each body of material are determined, in this method as in II, III, IV, VI, on the assumption that the volume per running foot, of a station solid, is constant; and in the case of the problem of Fig. 19 this is not true. It is believed that because of the uncertainties as to swell and distribution, the limits of error in the cross-sectioning, and because of the small cost of one station-yard of haul, and of the extra work involved in computing overhaul by that method, the use of Method VIII will not be justified except under extraordinary conditions.

To sum up: The percentage errors in the results obtained by Methods II, III, IV, VI, and VIII show the true relative accuracies of those methods for all profiles similar to that of Fig. 19. For bodies of material of other shapes the relative accuracies will be different.

The percentage of error in the result obtained by Method I is dependent entirely upon the skill and judgment of the computer. If, when about to compute by some one of the more accurate methods the overhaul for the various bodies of material along a given profile, the computer will take the trouble to compute first by eye (Method I) the overhaul on each body and make a record thereof, he will have a rough check on the correctness of the later results obtained by the more accurate method and at the same time he will have an opportunity to find the percentage of error in each of the results obtained by eye, and thus find approximately his limit of error in the use of that method.

The error in the result obtained by Method V is a resultant of the error obtained by Method IV and the errors incidental to plotting and scaling the mass curve, and the latter errors depend upon the skill of the draftsman and the scale of the mass curve. In the same way, Method VII gives an error in any case, which is the resultant of the error coming from Method VI and the errors of plotting and scaling the mass curve. See Sec. 203 for comments on results obtained for the overhaul of Fig. 38.

It is suggested that the engineer who is responsible for the selection of a method for computing overhaul on an extensive piece of earthwork, apply Methods IV, V, VI, and VII to a few of those bodies of overhauled material for which the four methods may be expected to give the most widely differing results, and select for the computation of the remaining overhaul that method which, while giving satisfactorily precise results, requires the least work. When time permits, Methods V and VII serve as excellent checks on Methods IV and VI respectively. The engineer will usually be compelled by lack of time to use Method I to check the overhaul computed by subordinates by some one of the more exact methods.

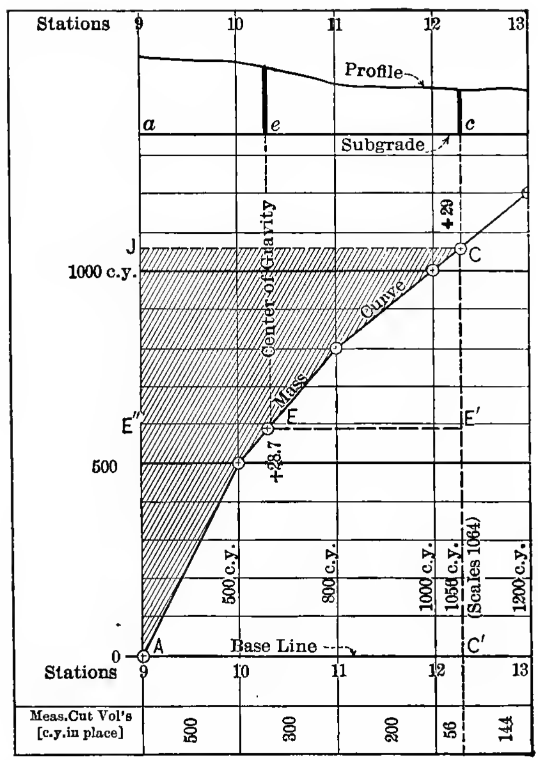


FIG. 24.

FIND CENTER OF GRAVITY OF CUT *ac*: The distance $E'E$ from the ordinate $C'C$ to the vertical Ee which contains the center of gravity of cut *ac* is (Sec. 34)

$$E'E = \frac{\text{area } AECC' \text{ (sta-yds.)}}{\text{ordinate } C'C \text{ (c.y.)}}$$

CHAPTER VII

OVERHAUL COMPUTED FOR THE COMPLEX CASE OF FIG. 38

In this chapter Methods I, IV, V, VI, and VII of overhaul computation are applied, one after another, to the complex problem of Fig. 38. The steps in each method are formally stated. The results are presented in Fig. 36 on the lines beginning with "B," and compared in Sec. 203. This chapter, like the preceding, is designed to serve as a working guide to the computer as soon as the method has been selected. (Fig. 36 faces page 67; Figs. 36a, 37, and 38 face page 120.)

147. Each of the overhaul computation methods, I, IV, V, VI, and VII (Secs. 47–55), will be applied in this chapter to the complex overhaul problem presented by the profile of Fig. 38.

It will be observed that the problem of Fig. 38 differs from that of Fig. 19 in two points: (1) In Fig. 38 there are four distinct bodies of material overhauled, while in Fig. 19 there is but one. (2) The work of determining swell- and equating-factors and distribution of material was simple in Fig. 19, but is comparatively complicated in Fig. 38. After we have determined the swell- and equating-factors and the distribution of material, for the case of Fig. 38, we shall find that the further steps of determining limits, centers of gravity, etc., are little more than repetition of the corresponding steps taken in Chapter VI for Fig. 19.

To conform to good practice, we first make ready for the computations by gathering all the available data. All the data for the overhaul problem in hand are given in Sec. 148, which is Step I of Method I, and are repeated in Sec. 157.

The results obtained by the different methods of computation will be recorded in Fig. 36, and compared in Sec. 203.

Overhaul of Fig. 38 Computed by Method I

(In Method I center of gravity and all limits are determined by eye.)

148. **Step 1. Data.** — We have the following data bearing on the distribution of material between t_1 and t_{17} in Fig. 37 which shows a small-scale copy of the profile of Fig. 38.

(1) All the grading was done with a steam-shovel plant.

(2) Volumes computed from final measurements are as follows:

Fill.	C.y.	Cut.	C.y.
t_1t_2	19,485	t_2t_4	28,743
t_4t_7	35,423	t_7t_8	14,899
t_8t_9	23,312	t_9t_{12}	29,183
$t_{12}t_{14}$	35,385	$t_{14}t_{16}$	29,063
$t_{16}t_{17}$	4,600		

The volumes tabulated above are shown on Fig. 37; and the volumes station by station are shown on Fig. 38.

(3) The grading was performed in this order: (a) The shovel went first to cut $t_{14}t_{16}$. The excavated material was hauled in both directions, making all of fill $t_{16}t_{17}$ and nearly all of fill $t_{12}t_{14}$. After completing this cut (b) the shovel moved to t_{12} and made a cutting to t_9, sending part of the material to the right, completing fill $t_{12}t_{14}$; part to fill t_8t_9; and a third part to fill t_4t_7. Without completing cut t_9t_{12} (c) the shovel moved to t_8 and made a cutting from t_8 to t_7, sending the material to fill t_4t_7. The shovel then turned around and made a cutting from t_7 to t_8, sending the material to fill t_4t_7. This work completed the cut t_7t_8. (d) The shovel moved to t_9 and completed the cut t_9t_{12}, sending a part of the material to fill t_8t_9 — enough to complete that fill — and the remainder of the material to fill t_4t_7. (e) The shovel then turned around and moved to t_4, and excavated cut t_2t_4, a part of the material making the entire fill t_1t_2, and the remainder completing fill t_4t_7.

(4) From diary and daily car records the number of car-loads hauled from cut t_9t_{12} to fill t_4t_7 was ascertained. From the monthly

estimates and car records the average car-load (cubic yards in place) was computed. In this way it was computed that 5000 c.y. (in place) of cut t_9t_{12} went to fill t_4t_7.

(5) From field diaries and car records it appeared that 1400 c.y. (in place) went from cut t_9t_{12} to fill $t_{12}t_{14}$.

(6) Swell. Cut $t_{14}t_{16}$ had a uniform swell. The 1400 c.y. hauled from cut t_9t_{12} to fill $t_{12}t_{14}$ had no swell. The material hauled to fill t_8t_9 had a uniform swell. The materials making fills t_1t_2 and t_4t_7 had the same swell. These swells were based on the judgment of those who watched the progress of the grading. No measurements had been made from which the swell of any one body of material could be directly computed.

(7) Free-haul distance is 1000 ft. (Contract.)

(8) The American Railway Engineering Association basis of computing overhaul is to be used. (Instructions.)

149. Step 2. Distribution of material. — The data (Sec. 148) show that the distribution of material was complex (Secs. 2 and 3). It is impossible to compute the overhaul on the basis of actual distribution, so we substitute for the actual, a simple, ideal distribution. The arrows and vertical division lines of Fig. 36a, which is a small copy of the profile of Fig. 38, show the ideal distribution adopted. (In Fig. 36a the vertical division lines are placed in position by eye, with no time wasted on trying to get true proportions. In fact the originals of Figs. 36a and 37 were merely freehand sketches upon which to arrange in order the general distribution data.)

150. Step 3. Free-haul limits and other limits of bodies of material. — The free-haul distance is 1000 ft. (Sec. 148 (7)). The free-haul limits were picked out by eye, and marked in pencil on the profile (Sec. 42). For example, the free-haul limits adjacent to the grade point, 72 + 00 (Fig. 38), were taken to be 66 + 80 and 76 + 80, because to the eye of the estimator the cut and fill appeared to balance between those two points. Likewise it appeared when looking at the fill on the extreme right of the profile that this would require all the material of the adjacent cut back as far as 81 + 70. The limits of bodies of material which appeared to be overhauled have been entered in Fig. 36 on lines B–I–1, 3, 4. (Such formal record may be considered superfluous in practice for the details of rough estimates of overhaul. The details are entered here to permit comparisons to be made between the various methods of computing overhaul.)

151. **Step 4. Centers of gravity.** — The center of gravity of each body of cut and of fill overhauled was picked out by eye (Sec. 27). For example, the center of gravity of cut, 76 + 80 to 81 + 70, appeared to the estimator to be at 78 + 90. (To make the best estimate of the position of the center of gravity of a body of cut or of fill, the profile to the right and to the left of that body should be concealed from view while making the estimate.) The centers of gravity, determined by eye, have been entered in Fig. 36, on lines B–I–1, 3, 4.

152. **Step 5. Average haul-distance.** — The average haul-distance for a body of overhauled material is the distance from its center of gravity in cut to its center of gravity in fill. Thus the average haul-distance for the material of cut, 76 + 80 to 81 + 70, is the distance between sta. 78 + 90 (column 6) and sta. 63 + 50 (column 10), or 15.4 stas., which is entered in column 11.

153. **Step 6. Average overhaul-distance.**—The average overhaul-distance for any body of overhauled material is equal to the average haul-distance for that body less the free-haul distance. The free-haul distance is given as 1000 ft. (Sec. 148 (7)). Thus the average overhaul-distance for the material of cut, 76 + 80 to 81 + 70, is equal to 15.4 (taken from column 11) less 10 = 5.4 stas., which is entered in column 12.

154. **Step 7. Volumes of bodies of overhauled material.** — If the station-volumes along the profile are given, the volume of each body of overhauled material may be found by summing

the station-volumes that constitute that body. In this way the volumes recorded in column 15, lines B–I–1, 3, 4, were found. If the station-volumes are not included with the data, they may be estimated with the help of earthwork tables or a yardage scale.

155. Step 8. The overhaul. — The overhaul on a body of material is equal to the volume (in place) of that material multiplied by its average overhaul-distance. For example, the overhaul on the material of cut, 76 + 80 to 81 + 70, is equal to 21,900 (taken from column 15, Fig. 36, line B–I–4) multiplied by 5.4 (taken from column 12) = 118,260 sta-yds., which is entered in column 18.

156. Step 9. Statement of overhaul. — The full statement of the overhaul just estimated for the profile, Fig. 38, is given on the upper set of lines B–I–1, 3, 4, Fig. 36. The lower set of lines B–I–1, 3, 4 shows results obtained by a second estimator.

Overhaul of Fig. 38 Computed by Method IV

(In Method IV center of gravity of body of cut or fill is assumed to lie at its center of volume, and is computed by arithmetic. Limits are computed by arithmetic.)

157. Step 1. Data. — We have the following data bearing on the distribution of material between t_1 and t_{17} in Fig. 37 which shows a small-scale copy of the profile of Fig. 38:

(1) All the grading was done with a steam-shovel plant.

(2) Volumes computed from final measurements are as follows:

Fill.	C.y.	Cut.	C.y.
t_1t_2	19,485	t_2t_4	28,743
t_4t_7	35,423	t_7t_8	14,899
t_8t_9	23,312	t_9t_{12}	29,183
$t_{12}t_{14}$	35,385	$t_{14}t_{16}$	29,063
$t_{16}t_{17}$	4,600		

The volumes tabulated above are shown on Fig. 37; and the volumes station by station are shown on Fig. 38.

(3) The grading was performed in this order: (a) The shovel

went first to cut $t_{14}t_{16}$. The excavated material was hauled in both directions, making all of fill $t_{16}t_{17}$ and nearly all of fill $t_{12}t_{14}$. After completing this cut (b) the shovel moved to t_{12} and made a cutting to t_9, sending part of the material to the right, completing fill $t_{12}t_{14}$; part to fill t_8t_9; and a third part to fill t_4t_7. Without completing cut t_9t_{12} (c) the shovel moved to t_8 and made a cutting from t_8 to t_7, sending the material to fill t_4t_7. The shovel then turned around and made a cutting from t_7 to t_8, sending the material to fill t_4t_7. This work completed the cut t_7t_8. (d) The shovel moved to t_9 and completed the cut t_9t_{12}, sending a part of the material to fill t_8t_9 — enough to complete that fill — and the remainder of the material to fill t_4t_7. (e) The shovel then turned around and moved to t_4, and excavated cut t_2t_4, a part of the material making the entire fill t_1t_2, and the remainder completing fill t_4t_7.

(4) From diary and daily car records the number of car-loads hauled from cut t_9t_{12} to fill t_4t_7 was ascertained. From the monthly estimates and car records the average car-load (cubic yards in place) was computed. In this way it was computed that 5000 c.y. (in place) of cut t_9t_{12} went to fill t_4t_7.

(5) From field diaries and car records it appeared that 1400 c.y. (in place) went from cut t_9t_{12} to fill $t_{12}t_{14}$.

(6) Swell. Cut $t_{14}t_{16}$ had a uniform swell. The 1400 c.y. hauled from cut t_9t_{12} to fill $t_{12}t_{14}$ had no swell. The material hauled to fill t_8t_9 had a uniform swell. The materials making fills t_1t_2 and t_4t_7 had the same swell. These swells were based on the judgment of those who watched the progress of the grading. No measurements had been made from which the swell of any one body of material could be directly computed.

(7) Free-haul distance is 1000 ft. (Contract.)

(8) The American Railway Engineering Association basis of computing overhaul is to be used. (Instructions.)

158. Step 2. Distribution of material. — The data show that the distribution of the material was complex (Secs. 1, 2). It was impossible to compute the overhaul on the basis of the actual distribution, and accordingly a simple ideal scheme of distribution has been substituted for the actual. The arrows and vertical division-lines of Fig. 36a, which is a small-scale copy of the profile of Fig. 38, show the ideal distribution adopted. (The original of Fig. 36a was merely a freehand sketch which was made to serve as a diagram on which to hang in orderly array the general data on distribution.) The bodies of cut resulting from the ideal plan of distribution adopted are marked C_1, C_2, C_3, . . . , and the bodies of fill are marked F_1, F_2, F_3, . . . , in Figs. 36a and 37. The limits of bodies of material have been marked t_1, t_2, t_3, . . . t_{17} in Figs. 36a and 37. It remains to compute the station and plus of each of the limits, and this is done in Sec. 160.

159. Step 3. Determination of swell-factors and volumes of the several bodies of cut and of fill. — As it had been found impracticable to make field measurements during the progress of the work sufficient to determine directly the swell-factor of any one body of cut, it was necessary to estimate the swell-factor of each body of cut, basing the estimate on the data at hand; and this was done in the following manner:

Let $C_1, C_2, C_3, \ldots$ = volumes of cuts $C_1, C_2, C_3, \ldots$ respectively;
$s_1, s_2, s_3, \ldots$ = swell-factors of cuts $C_1, C_2, C_3, \ldots$ respectively;
$F_1, F_2, F_3, \ldots$ = volumes of fills $F_1, F_2, F_3, \ldots$ respectively;
$q_1, q_2, q_3, \ldots$ = equating-factors of fills $F_1, F_2, F_3, \ldots$ respectively.

These symbols are now entered in Fig. 36a.

NOTE. — In this work the result of each step or partial step should be immediately entered on the profile in correct position. In this way confusion is avoided and the chance of blunder is reduced to a minimum.

As stated in Sec. 157 (3), all the material from cuts C_1, C_2, C_3, and C_5 just made the fills F_1, F_2, F_3, and F_4. Therefore

$$C_1s_1 + C_2s_2 + C_3s_3 + C_5s_5 = F_1 + F_2 + F_3 + F_4.$$

From Sec. 157 (6), $s_1 = s_2 = s_3 = s_5$;

therefore, $$s_1 = \frac{F_1 + F_2 + F_3 + F_4}{C_1 + C_2 + C_3 + C_5}.$$

From the data, Sec. 157 (2), we have

$$F_1 = 19{,}485 \text{ c.y.} \qquad C_1+C_2 = 28{,}743 \text{ c.y.}$$
$$F_2+F_3+F_4 = 35{,}423 \text{ c.y.} \qquad C_3 = 14{,}899 \text{ c.y.}$$
$$F_1+F_2+F_3+F_4 = 54{,}908 \text{ c.y.} \qquad C_5 = 5{,}000 \text{ c.y.}$$
$$C_1+C_2+C_3+C_5 = 48{,}642 \text{ c.y.}$$

Therefore

$$s_1 = \frac{54{,}908}{48{,}642} = 1.128,$$

and s_2, s_3, and s_5 have the same value. The corresponding equating-factor for fills F_1, F_2, F_3, and F_4 is

$$q_1 = q_2 = q_3 = q_4 = \frac{1}{s_1} = \frac{1}{1.128} = 0.886.$$

$$\text{Now } C_1 = F_1q_1 = 19{,}485 \times 0.886 = 17{,}250 \text{ c.y.}$$
$$C_2 = (C_1 + C_2) - C_1 = 28{,}743 - 17{,}250 = 11{,}493 \text{ c.}$$
$$F_2 = C_2s_2 = 11{,}493 \times 1.128 = 12{,}960 \text{ c.y.}$$
$$F_3 = C_5s_5 = 5000 \times 1.128 = 5640 \text{ c.y.}$$
$$F_4 = (F_2 + F_3 + F_4) - (F_2 + F_3)$$
$$= 35{,}423 - (12{,}960 + 5640) = 16{,}823 \text{ c.y.}$$

As a check,

$$F_4 = C_3s_3 = 14{,}899 \times 1.128 = 16{,}805 \text{ c.y.}$$

This checks the preceding value of F_4 within 18 c.y., and this discrepancy is no greater than may be reasonably expected from the use of the slide-rule; hence the check is considered to be satisfactory. These quantities are entered on Fig. 37 as soon as computed.

The swell-factor for cut C_6 is unity (Sec. 157 (6)); therefore

$$F_6 = C_6 s_6 = 1400 \times 1 = 1400 \text{ c.y.}$$
$$C_4 = (C_4 + C_5 + C_6) - (C_5 + C_6)$$
$$= 29{,}183 - (5000 + 1400) = 22{,}783 \text{ c.y.}$$

The equating-factor for fill F_5 is $q_5 = \frac{C_4}{F_5} = \frac{22{,}783}{23{,}312} = 0.977$.

The volume of fill $F_7 = (F_6 + F_7) - F_6 = 35{,}385 - 1400 = 33{,}985$ c.y.

The equating-factors for fills F_7 and F_8 are found thus: From the data, Sec. 157 (6), and the foregoing computations we have

$$F_7 q_7 + F_8 q_8 = C_7 + C_8,$$

and

$$q_7 = q_8.$$

Hence

$$q_7 = q_8 = \frac{(C_7 + C_8)}{(F_7 + F_8)} = \frac{29{,}063}{(33{,}985 + 4600)} = 0.753.$$

$$C_7 = F_7 q_7 = 33{,}985 \times 0.753 = 25{,}600 \text{ c.y.}$$
$$C_8 = F_8 q_8 = 4600 \times 0.753 = 3{,}460 \text{ c.y.}$$
$$C_7 + C_8 = 29{,}060 \text{ c.y.}$$

This value of $C_7 + C_8$ differs from 29,063 (the volume given in Step 1 (2)) by 3 yards. This is a close agreement.

The swell-factors of cuts C_7 and C_8 are

$$s_7 = s_8 = \frac{1}{q_7} = \frac{1}{q_8} = \frac{1}{0.753} = 1.328.$$

Having entered on the profile, Fig. 37, all these computed volumes, swell-factors, and equating-factors, we are ready for the next Step.

160. **Step 4. Limits of bodies of material.** — Figs. 37 and 38. There is no division plane in fill F_1. The position of division plane t_3 which limits cut C_1 on the right is found thus: We have found that $C_1 = 17{,}250$ c.y.; by the method given in Sec. 20, we find that in passing from t_2 (sta. 14 + 12) to sta. 17 + 30.5 we have a total volume of 14,698; and that in passing from t_2 to

18 + 05.5 we have a total of 14,698 + 5613 = 20,311 c.y. Hence the right-hand limit t_3 of cut C_1 must lie between 17+30.5 and 18 + 05.5. Now between these two stations the volume per running foot =

$$\frac{5613}{(\text{sta. } 18 + 05.5) - (\text{sta. } 17 + 30.5)} = \frac{5613}{75} = 74.8 \text{ c.y.}$$

Therefore t_3 lies to the right of 17 + 30.5 a distance = $\frac{(17,250 - 14,698)}{74.8} = 34.1$ ft. Therefore the division plane between cut C_1 and cut C_2 lies at

$$t_3 = (\text{sta. } 17 + 30.5) + 34.1 = \text{sta. } 17 + 64.6.$$

(To check this result we compute the distance from sta. 18 + 05.5 back to t_3: t_3 to 18 + 05.5 = $\frac{(20,311 - 17,250)}{74.8} = 40.9$. Therefore the station of t_3 is (sta. 18 + 05.5) − 40.9 = sta. 17+64.6. This agrees with the preceding result. As the computations were performed with the slide-rule a discrepancy of some tenths of a foot is to be expected, and the precise agreement here is accidental.)

The details of the computations performed to ascertain the station and plus of each of the other division-planes are omitted. The results of the computations are tabulated below.

POSITIONS OF DIVISION PLANES.

Division plane.	Station and plus.	Substation volumes.	
		On the left.	On the right.
t_3	17+64.6	2552	3061
t_5	23+44.1	1180	5300
t_6	24+01.9	6180	300
t_{10}	52+97.8	3171	83
t_{11}	54+93.9	776	417
t_{13}	58+12.2	400	2180
t_{15}	81+24.1	530	1666

The position of each division plane is marked on Fig. 37, as soon as computed, and the station and plus entered thereon. Likewise the two parts, into which each division-line divides the station-volume which it cuts, are entered on the profile, Fig. 38.

Fig. 37 now carries all the data required for the next Step.

161. Step 5. Equate each station-volume of fill to volume in place. — This is done by multiplying each station-volume of fill by the proper equating-factor. (The ten-inch slide rule serves for this.) We multiply each station-volume of fills F_1, F_2, F_3, and F_4 by 0.886 (Step 3), and enter the resulting volumes in place, on the profile of Fig. 38. We multiply each station-volume of fill F_6 by 1, and enter the products on the profile. See Fig. 38. The station-volumes of fills F_7 and F_8 are each multiplied by 0.753; and the resulting volumes in place are entered on the profile.

The profile of Fig. 38 now shows the volume of each station (and partial station) in terms of cubic yards in place; and from now on we shall use only the cut-volumes and equated fill-volumes.

As a check on the computations of this Step, the sum of the equated station-volumes of each body of fill should be compared with the volume of the corresponding body of cut, to which it should be practically equal. Some discrepancy is to be expected owing to the uncertainty in the third figure of a product obtained by the use of the slide-rule. The discrepancy, if reasonably small, may be wiped out by arbitrarily correcting one or more of the equated station-volumes.

162. Step 6. Limits of free haul. — Use the method of Sec. 40. By this method the limits of free haul have been computed:

Stas. 25 + 12.4 and 35 + 12.4;
" 42 + 53.1 " 52 + 53.1;
" 66 + 72.9 " 76 + 72.9;

and these have been marked on the profile, Fig. 38. The sub-station-volume lying on each side of each limit has been computed and entered on the profile in its proper place. For example, the limit 76 + 72.9 divides the station-volume, 1615 c.y., lying between 76 + 50 and 77 + 00, into two parts, the left of which is 740 c.y., and the right, 875 c.y.

163. **Step 7. Centers of gravity.** — Use Method IV, Sec. 30. The centers of gravity, one in cut and the other in fill, have been computed by this method for each body of material overhauled. The results are entered in Fig. 36, lines B–IV–1, 2, 3, 4, columns 6 and 10. Thus the center of gravity of cut, 35 + 12.4 to 41 + 53.0, is 37 + 68.0 (column 6).

164. **Step 8. Average haul-distances.** — The average haul-distance for a body of material is the distance from its center of gravity in cut to its center of gravity in fill. For example, the average haul-distance for the body of cut, 35 + 12.4 to 41 + 53, is equal to (sta. 37 + 68.0) – (sta. 24 + 54.1) = 13.139 stas., which is entered in column 11, line B–IV–1, Fig. 36.

165. **Step 9. Average overhaul-distances.** — The average overhaul-distance for any body of overhauled material is equal to the average haul-distance for that body less the free-haul distance. The free-haul distance is 10 stas. (Sec. 157 (7)). Thus the average overhaul-distance for the material of cut, 35 + 12.4 to 41 + 53.0, is equal to 13.139 (taken from column 11) less 10 = 3.139 stas., which is entered in column 12.

166. **Step 10. The overhaul.** — The overhaul on each body of cut is equal to the volume of that cut multiplied by its average overhaul-distance. The volume of each body of cut overhauled is entered in column 15, and the corresponding overhaul-distance in column 12, on the same line; and the product of these two quantities is entered in column 18. Thus, the overhaul on the body of cut, 35 + 12.4 to 41 + 53.0, is 8990 × 3.139 = 28,220 sta-yds., which is entered in column 18, line B–IV–1, of Fig. 36.

167. **Step 11. Statement of overhaul.** — Full statement of the overhaul, computed above for the profile of Fig. 38, is entered on lines B–IV–1, 2, 3, 4 of Fig. 36.

Overhaul of Fig. 38 Computed by Method V

(In Method V center of gravity of body of cut or fill is assumed to lie at its center of volume, and is determined by mass curve. Limits are determined by mass curve.)

168. **Step 1. Data.** — Read Sec. 157.

169. **Step 2. Distribution of material.** — See Sec. 158.

170. **Step 3. Determination of swell-factors and volumes of the several bodies of cut and fill.** — Read Sec. 159.

171. **Step 4. Equate each station-volume of fill to volume in place.** — See Sec. 161. Equated fill-volumes only are used with the cut-volumes throughout the remaining steps of this Method.

172. **Step 5. Plot the mass curve.** — The origin of the mass curve was taken above the profile at sta. 10 + 08, Fig. 38. The cut-volumes are considered positive and the fill-volumes negative. The ordinate at each station is the algebraic sum of the station-volumes lying between that station and 10 + 08. The increment to the mass curve for a station-interval which contains both cut and fill, is the excess of the one over the other. In Fig. 38 the horizontal scale of the profile and mass curve is 1 in. = 400 ft.; the vertical scale of the profile is 1 in. = 20 ft.; and the vertical scale of the mass curve is 1 in. = 10,000 c.y.

NOTE. — The original drawing for this mass curve was made on a roll of cross-section paper, 10 × 10 to the inch; and the scales were 1 in. = 100 ft., horizontal, and 1 in. = 2500 c.y., vertical. The results given in the following Steps were obtained from the original drawing.

173. **Step 6. Limits of bodies of material.** — The base line of this mass curve is the balancing line. The points at which the mass curve cuts the balancing line mark the limits of the bodies of cut and fill. These points are noted and the plus of each is scaled off and entered on the mass curve. Thus there are division planes at stas. 17 + 64, 23 + 44, 54 + 94, 58 + 12, and 81 + 24. Also the horizontal drawn through the summit of the mass curve at sta. 41 + 53, is a balancing line, which, by

cutting the mass curve, gives us division planes at stas. 24 + 02, 41 + 53, and 52 + 98.

174. **Step 7. Limits of free haul.** — Find the limits of free haul by the method of Sec. 41. The plus of each limit of free haul is scaled and entered on the mass curve. The free-haul limits determined by this method are

Stas. 25 + 12 and 35 + 12;
" 42 + 53 " 52 + 53;
" 66 + 73 " 76 + 73.

175. **Step 8. Centers of gravity.** — The center of gravity of each body of material is found in cut and in fill by Method V, Sec. 31. Thus the center of gravity of cut, 35 + 12 to 41 + 53, is found to be at 37 + 66, which is entered in column 6, line B–V–1, Fig. 36. Centers of gravity thus determined are indicated above the mass curve, on Fig. 38, by the prefix "CM."

176. **Step 9. Average haul-distances.** — The average haul-distance for a body of material is the distance from its center of gravity in cut to its center of gravity in fill. Thus the average haul-distance for the body of cut, 35 + 12 to 41 + 53, is equal to (sta. 37 + 66) – (sta. 24 + 53) = 13.13 stas., which is recorded in column 11, line B–V–1, Fig. 36.

177. **Step 10. Average overhaul-distances.** — The average overhaul-distance for any body of overhauled material is equal to the average haul-distance for that body less the free-haul distance. The free-haul distance is 10 stas. (Sec. 148 (7)). Thus the average overhaul-distance for the material of cut, 35 + 12 to 41 + 53, is equal to 13.13 (taken from column 11) less 10 = 3.13 stas., which is recorded in column 12.

178. **Step 11. The overhaul.** — The overhaul on each body of cut is equal to the volume of that cut multiplied by its average haul-distance. The volume of each body of cut overhauled is entered in column 15, and the corresponding average overhaul-distance in column 12 on the same line; and the product of

these two quantities is entered in column 18. Thus the overhaul on the body of cut, 35 + 12 to 41 + 53, is 9000 × 3.13 = 28,170 sta-yds.

179. Step 12. Statement of overhaul. — The full statement of the overhaul of Fig. 38, as computed by Method V, is entered in Fig. 36, on lines B–V–1, 2, 3, 4.

Overhaul of Fig. 38 Computed by Method VI

(In Method VI the center of gravity of a body of cut or fill is computed arithmetically by the method of moments; the center of gravity of each station-volume being assumed to lie at its center of length. The limits are computed by arithmetic.)

180. Step 1. Data. — Read Sec. 157.

181. Step 2. Distribution of material. — Read Sec. 158.

182. Step 3. Determination of the swell-factors and of the volumes of the several bodies of cut and fill. — Read Sec. 159.

183. Step 4. Limits of bodies of material. — Read Sec. 160.

184. Step 5. Equate each station-volume of fill to volume in place. — Read Sec. 161.

185. Step 6. Free-haul limits. — By the method of Sec. 40 the free-haul limits are found to be as follows:

Stas. 25 + 12.4 and 35 + 12.4;
" 42 + 53.1 " 52 + 53.1;
" 66 + 72.9 " 76 + 72.9;

and these have been marked on the profile, Fig. 38. The sub-station-volume lying on each side of each limit has been computed and entered on the profile in its proper place. For example, the limit 25 + 12.4 divides the volume lying between stas. 25 + 00 and 26 + 00 into two parts, the left of which is 556 c.y., and the right, 3924 c.y.

186. Step 7. Centers of gravity. — By Method VI, Sec. 32, the center of gravity of each body of cut and of each body of fill is found. The centers of gravity thus computed have been entered on lines B–VI–1, 2, 3, 4 of Fig. 36 — for the cuts, in column 6; for the fills, in column 10. The computations made to find the position of the center of gravity of the body of cut lying between stas. 76 + 72.9 and 81 + 24.1 are given below.

Station.	g	V	a	Va	b	Vb
76+72.9	76+86.4	875	.864	756	5.136	4,494
77	77+25	2223	1.25	2,779	4.75	10,559
77+50	77+75	2663	1.75	4,660	4.25	11,318
78	78+50	5542	2.50	13,855	3.50	19,397
79	79+30	3327	3.30	10,979	2.70	8,983
79+60	79+80	2052	3.80	7,798	2.20	4,514
80	80+50	3941	4.50	17,735	1.50	5,911
81	81+12	530	5.12	2,714	.88	466
81+24.1						
Totals..	....	21,153 c.y.	....	61,276 sta-yds.	....	65,642 sta-yds.

g = center of gravity of station-volume on assumption that center of gravity lies at the mid-point.
V = station-volume in cubic yards.
a = length in stations, of lever-arm = distance from each mid-point, in turn, to the center of moments, sta. 76 + 00.
Va = moment (station-yards) of V about sta. 76 + 00.
b = lever-arm length, in stations = distance from each mid-point, in turn, to the center of moments, sta. 82 + 00.
Vb = moment (station-yards) of V about sta. 82 + 00.

Average lever-arm about sta. 76 + 00 = distance from center of gravity of cut to sta. 76 + 00 $\Big\} = \dfrac{61{,}276}{21{,}153} = 2.897$ stas.

Hence the center of gravity of the cut, sta. 76 + 72 to sta. 81 + 24.1, is at (sta. 76 + 00) + 2.897 stas. = sta. 78 + 89.7. This result may be checked in the following way.

Average lever-arm about sta. 82 + 00 = distance from center of gravity of cut to sta. 82 + 00 $\Big\} = \dfrac{65{,}642}{21{,}153} = 3.103$ stas.

Hence the center of gravity of the cut, sta. 76 + 72 to sta. 81 + 24.1, is at (sta. 82 + 00) − 3.103 stas. = sta. 78 + 89.7 (check). This result, sta. 78 + 89.7, is entered in column 6, line B–VI–4,

Fig. 36. The other centers of gravity were computed in like manner, and entered on lines B–VI, in columns 6 and 19, Fig. 36.

NOTE 1. — The computer who found the results above, multiplied and divided in the long way. Another computer, using the 10-inch slide-rule, on the same data found the sum of column *Va* to be 61,270, and of column *Vb*, 65,660; and found the center of gravity from moments *Va* to be at 78 + 90, and from moments *Vb*, at 78 + 89.8.

NOTE 2. — It will be observed that in the computation above, the center of moments was arbitrarily taken at 76 + 00 instead of 76 + 72.9 which is the left limit of the body of material under consideration, because of the greater ease thereby afforded in making the subtractions to obtain lengths of lever-arms, column *a*. For the same reason the center of moments for the check computations was taken at 82 + 00 rather than at 81 + 24.1.

187. Step 8. Average haul-distances. — We have already entered in Fig. 36, lines B–VI–1, 2, 3, 4, the center of gravity of each body of cut overhauled, and the center of gravity of the corresponding body of fill. The distance between the two centers of gravity entered on the same line is the average haul-distance for the body of cut, the limits of which appear on that line, and is entered in column 11 on the same line. Thus the average haul-distance for the body of cut lying between the limits 52 + 53.1 and 52 + 97.8 is (sta. 52 + 75.5) – (sta. 42 + 25.2) = 10.503 stas.

188. Step 9. Average overhaul-distances. — The average overhaul-distance for each body of cut is equal to the average haul-distance of the body less the free-haul distance (1000 feet in this case). The average overhaul-distance for each body of cut is entered in column 12, Fig. 36.

189. Step 10. The overhaul. — The volume of each body of cut is entered in column 15, Fig. 36; and the corresponding overhaul, which is the product of the volume by the average overhaul-distance, is entered in column 18.

190. Step 11. Statement of overhaul. — Lines B–VI–1, 2, 3, 4, of Fig. 36, now constitute a full statement of the overhaul on the material represented by the profile of Figs. 37 and 38, as computed by Method VI.

Overhaul of Fig. 38 Computed by Method VII

(In Method VII the center of gravity of each body of cut or fill is computed by means of the mass curve by the method of moments. All limits are determined by the mass curve.)

191. Step 1. **Data.** — Read Sec. 157.

192. Step 2. **Distribution of material.** — Read Sec. 158.

193. Step 3. **Determination of the swell-factors and the volumes of the several bodies of cut and fill.** — Read Sec. 159.

194. Step 4. **Equate each station-volume of fill to volume in place.** — Read Sec. 161. Equated fill-volumes only are used with cut-volumes throughout the remaining Steps of this Method.

195. Step 5. **Plot the mass curve.** — The origin for the mass curve was taken above the left end of the profile — at sta. 10+08, Fig. 38. The cut-volumes are reckoned positive and the fill-volumes negative. The ordinate at each station is the algebraic sum of the station-volumes lying between that station and sta. 10 + 08. In Fig. 38 the horizontal scale of the profile and of the mass curve is 1 inch = 400 feet; the vertical scale of the profile is 1 inch = 20 feet; and the vertical scale of the mass curve is 1 inch = 10,000 c.y.

NOTE. — The original drawing for this mass curve was made on a roll of cross-section paper, 10 × 10 to the inch; and the scales were: horizontal, 1 inch = 100 feet; vertical, 1 inch = 2500 c.y. The results given in the following Steps were obtained from the original drawing.

196. Step 6. **Limits of bodies of material.** — The base line of this mass curve is a balancing line. The points at which the mass curve cuts the balancing line, mark the limits of the bodies of cut and of fill. These points are noted and the plus of each is scaled off and entered on the mass curve. Thus there are division planes at stas. 17 + 64; 23 + 44; 54 + 94; 58 + 12; 81 + 24. Also the horizontal drawn through the summit of the mass curve at 41 + 53 is a balancing line, which, by cutting the

mass curve, gives us division planes at 24 + 02; 41 + 53; 52 + 98.

197. Step 7. Limits of free haul. — By the method of Sec. 41 we find the free-haul limits. The plus of each limit of free haul is scaled, and entered on the mass curve. There are free-haul limits at stas. 25 + 12 and 35 + 12; at stas. 42 + 53 and 52 + 53; and at stas. 66 + 73 and 76 + 73.

198. Step 8. Centers of gravity. — The center of gravity of each body of material overhauled was found by Method VII, Sec. 34, and entered just below the mass curve (Fig. 38) and marked by a vertical arrow. The center of gravity of cut of which the limits are 35 + 12 and 41 + 53 is found to be at sta. 37 + 92. The center of gravity of fill, 24 + 02 to 25 + 12, is found to lie at sta. 24 + 55. The centers of gravity of these and of the other bodies of material have been entered on lines B–VII–1, 2, 3, 4 in columns 6 and 10, Fig. 36.

199. Step 9. Average haul-distances. — The average haul-distances were computed and entered in column 11, Fig. 36. The average haul-distance for material taken from cut, 35 + 12 to 41 + 53, to fill, 24 + 02 to 25 + 12, is the difference between the station and plus of column 6 and the station and plus of column 10, of line B–VII–1. This difference, 13.37 stas., is entered in column 11.

200. Step 10. Average overhaul-distances. — The average haul-distance for each body of cut is entered in column 12, Fig. 36. It will be observed that each distance in column 12 is equal to the corresponding distance in column 11 less the free-haul distance of 10 stas.

201. Step 11. The overhaul. — The overhaul on each body of cut overhauled is equal to the product of volume of cut (entered in column 15, Fig. 36) by the overhaul distance (entered in column 12). For example, the overhaul on cut, 35 + 12 to 41 + 53, is 9000 × 3.37 = 30,330 sta-yds.; and this is entered in column 18, on line B–VII–1.

202. **Step 12. Statement of overhaul.** — The complete statement of the overhaul on the profile shown in Figs. 36a, 37, 38, as computed by Method VII, is given on lines B–VII–1, 2, 3, 4 of Fig. 36.

Comparison of Results

203. **The foregoing results compared.** — The results obtained in this chapter are clearly set forth in Fig. 36, and are readily compared. However, to save the time of the reader the results have been retabulated in three different ways below. Method VIII, the most accurate of all, was not applied to Fig. 38, because of the labor involved. So in the comparisons of Tables I and II the results obtained by Method VI, which method stands second in point of accuracy (Sec. 146), are considered to be true results.

Referring to Table I. The errors in the results of Method I, in which the limits and centers of gravity are determined by eye directly from the profile, range from − 13% to + 5%, approximately. Observe that the error in the total overhaul is only about 6%; less than 13%, as is to be expected because the error in the sum of several estimates will always be smaller than the largest error of the component estimates. In Fig. 36 are shown the results obtained by two men separately applying Method I. The group of results that we have not copied into the following tables has decidedly smaller errors than those copied. However, a third estimator might obtain results in error by more than 13%. It should be stated here that the profile of Fig. 38 presents extremely unfavorable conditions for determining overhaul by Method I, because of the fact that cuts (Fig. 36a) t_2t_4, t_9t_{12}, $t_{14}t_{16}$, and fills t_4t_7, t_8t_9, $t_{16}t_{17}$, were on steep hillsides. The fair results obtained by this, by far the quickest, method of computing overhaul, seem to indicate that it has decided value for making rough estimates of overhaul, whether for checking overhaul computed by some more accurate method or for preliminary studies; or, under proper conditions, for making monthly estimates of overhaul. The best results from this method are to be expected from one who is familiar with the ground along the profile, and with one of the more accurate methods of overhaul computation. For remarks on acquiring skill in the use of Method I, see Sec. 146.

TABLE I

Errors in the overhaul of Fig. 38 computed by Methods I, IV, V, VI, VII, grouped by Methods. Overhaul computed by Method VI is taken as the true overhaul.

Method.	Body of material, No.	Overhaul (sta-yds.).	Error (sta-yds.).	Error (per cent).	Range of error (per cent).
I.....	1	32,200	+ 1,607	+ 5.25	
	2	....	....	....	
	3	101,500	+ 1,035	+ 1.03	
	4	118,260	−18,057	−13.20	−13.20 to +5.25
	Total....	251,960	−16,270	− 6.06	
IV.....	1	28,220	− 2,373	− 7.75	
	2	835	− 20	− 2.34	
	3	100,205	− 260	− .26	− 0.26 to −7.75
	4	131,303	− 5,014	− 3.68	
	Total....	260,563	− 7,667	− 2.86	
V.....	1	28,170	− 2,423	− 7.92	
	2	833	− 22	− 2.57	
	3	100,250	− 215	− .21	− 0.21 to −7.92
	4	131,652	− 4,665	− 3.42	
	Total....	260,905	− 7,325	− 2.73	NOTE.—Overhaul computed by Method VI is taken as the true overhaul, in this table.
VI.....	1	30,593	0	0.00	
	2	855	0	0.00	
	3	100,465	0	0.00	
	4	136,317	0	0.00	
	Total....	268,230	0	0.00	
VII.....	1	30,330	− 263	− 0.86	
	2	850	− 5	− 0.58	
	3	100,450	− 15	− 0.015	− 0.86 to + 0.16
	4	136,528	+ 211	+ 0.16	
	Total....	268,158	− 72	− 0.027	

In Method IV the computations are all made by arithmetic; in Method V, by the mass curve; but in both the center of volume is taken as the center of gravity. On the individual bodies the error ranges from zero to − 8%, approximately; but the error of the total is only about 3%. On this particular profile the errors due to assuming that center of gravity lies at center of volume, are all of one sign, and therefore cumulative. Of course the sign of the error for any given body of material depends upon the shape of that body.

In Table II the results obtained by the several Methods for each body of material are grouped together.

TABLE II

Errors in the overhaul of Fig. 38 computed by Methods I, IV, V, VI, VII, grouped by bodies of overhauled material. Overhaul computed by Method VI is taken as the true overhaul.

Body of overhauled material, No.	Method of computation.	Overhaul (sta-yds.).	Error (sta-yds.).	Error (per cent).
1	I	32,200	+ 1,607	+ 5.25
	IV	28,220	− 2,373	− 7.75
	V	28,170	− 2,423	− 7.92
	VI	30,593	0	0.00
	VII	30,330	− 263	− 0.86
2	I	0	855	
	IV	835	− 20	− 2.34
	V	833	− 22	− 2.57
	VI	855	0	0.00
	VII	850	− 5	− 0.58
3	I	101,500	+ 1,035	+ 1.03
	IV	100,205	− 260	− 0.26
	V	100,250	− 215	− 0.21
	VI	100,465	0	0.00
	VII	100,450	− 15	− 0.015
4	I	118,260	− 18,057	− 13.20
	IV	131,303	− 5,014	− 3.68
	V	131,652	− 4,665	− 3.42
	VI	136,317	0	0.00
	VII	136,528	+ 211	+ 0.16

In Table III all the overhaul quantities obtained by mass curve are so arranged as to bring out clearly the range of errors which

TABLE III

Errors in overhaul computed by mass curve, due to errors in plotting and scaling.

Body of overhauled material, No.	Center of volume taken as center of gravity. Overhaul by Method—		Error = V–IV (sta-yds.).	Error (per cent).
	V.	IV.		
1	28,170	28,220	− 50	−0.18
2	833	835	− 2	−0.24
3	100,250	100,205	+ 45	+0.05
4	131,652	131,303	+349	+0.27
Total.....	260,905	260,563	+342	+0.13
	Center of gravity by moments. Overhaul by Method—		Error = VII–VI. (sta-yds.)	Error (per cent).
	VII.	VI.		
1	30,330	30,593	−263	−0.86
2	850	855	− 5	-0.58
3	100,450	100,465	− 15	-0.015
4	136,528	136,317	+211	+0.16
Total.....	268,158	268,230	− 72	−0.027

are due to using the mass curve in place of arithmetic. Taking the eight results, excluding totals, the errors range from − 0.86% to + 0.27%. It will be observed that the two large errors, 0.86% and 0.58%, occur with the smaller bodies of material, as we should expect, for the absolute error of plotting and scaling is independent of the distance (within one scale-length), and the relative error therefore decreases as the distance increases. Notwithstanding these large errors, the error in the sum for Method VII is only 0.027%. See Secs. 16 and 17. It should be remembered that these results were obtained from the original of the mass curve of Fig. 38, which original had a horizontal scale of 1 inch = 100 feet — a scale four times as large as that of Plate A profile paper — and a vertical scale of 1 inch = 2500 c.y. Computation will show that the limit of error in the overhaul obtained for body No. 4 by this large-scale mass curve, is about 0.5%, while the limit of error for the smaller body, No. 1, is about 1.2% — this on the assumption that all errors of plotting and scaling are of maximum size and of such sign as not to permit cancellation in summing them. Thus the small errors actually obtained are small because errors of maximum size are not frequent and moreover tend to cancel in the summation. The larger the scale the smaller the error to be expected.

When we consider the uncertainty in the volumes obtained by cross-sectioning irregular cuts and fills, and in the swell-factors, and, further, remember that the actual distribution of material must be replaced by an assumed distribution before we can proceed with the computation of overhaul, the use of the mass curve, when due regard is paid to the scale, seems, even for the method of moments, to be fully justified.

In any case the choice of method will be influenced by the quality of the data supplied to, and the time at the disposal of, the computer, the use to which the results are to be put, and finally by the previous training and experience of the computer.

PART II

ECONOMIC DISTRIBUTION OF MATERIAL ALONG THE PROFILE

(PART II IS DEVOTED EXCLUSIVELY TO FUTURE HAUL)

Explanation of signs used in Part II:

$<$ should be read "is less than;"
$\nless$ " " " "is not less than;"
$>$ " " " "is greater than;"
$\ngtr$ " " " "is not greater than."

CHAPTER VIII

PRELIMINARY CONSIDERATIONS

This chapter discusses the effect of swell on planning distribution, and limit of profitable haul; and states the principle of economic distribution.

204. **Swell-factor. Problems of past and future haul compared.** — It has been pointed out in Secs. 2 and 3 that in many cases it is impracticable to determine with exactness the swell which has resulted from moving a given body of material from cut to fill. Plainly, then, a more or less rough estimate of the swell which will result from future grading must be relied upon when planning distribution. The most reliable estimate of swell can be made only when the nature of the material to be moved and the method of excavating, hauling, and dumping to be used are known. In any case the engineer must be content to make the best estimate of swell that he can from the data at hand; make a tentative plan of distribution based on the estimated swell; and check or revise this plan from time to time —

from month to month, say — as he obtains additional data on swell from the grading in progress. Of course when any unexpected condition of material is found in the work of grading, the side slopes as well as the estimated swell-factors may require modification. Suppose that the original plan of distribution of material along the profile was based on quantities computed from an assumed side slope of 1 : 1 in cut and of 1.5 : 1 in fill. If at the end of the first month, say, of grading it is found that the material is such as to make it wise to change the side slopes in one or more places to such an extent as to make a considerable change in the estimated yardage on which the original plan was based, the engineer should revise the plan of distribution, using volume estimates based on the revised side slopes. Other conditions may arise which make it impracticable to carry out in every detail the best possible original plan of distribution. For example, heavy and continued floods may make it impossible to build a bridge in time to permit the passage of material as planned; or delay in purchase of a piece of land for right-of-way, waste bank, or borrow pit may prevent carrying out some detail of the original plan of disposition of material; or some condition not developed by the investigations preliminary to the making of the plan of distribution may even require a change of alinement or of grade which will upset the original plan of distribution, at least in part. The nature of the material in each proposed excavation can be (though seldom is) determined with all requisite accuracy by borings or test pits; but even then no exact estimate of resulting swell can be made, for the reasons stated in Secs. 2 and 3. The frequent checking and occasional revision of the plan of distribution takes much time, but is nevertheless imperative from the standpoint of economy. See Proceedings, American Railway Engineering Association, vol. 7 (1906), pp. 414–427.

205. Cut-volumes must be reduced to fill-volumes. — In planning the distribution of the material along the profile it is first necessary to convert the cut-volumes into swelled volumes by multiplying each cut-volume by its estimated swell-factor. We cannot equate fill-volumes to volumes in place at this point; for until the planning is completed and we have determined the distribution of material we do not know from what cut or part of a cut any given fill or part of fill is going to be made, and therefore do not know what equating-factor to use in any given

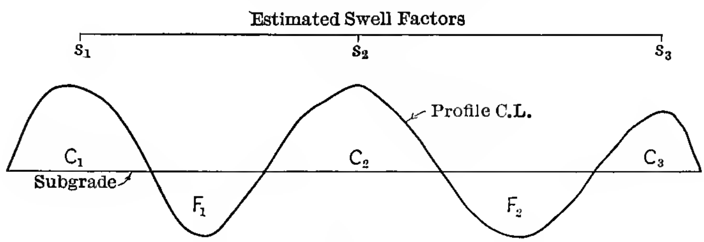


FIG. 40.

case. For example, in planning the distribution of the material shown in Fig. 40 we can say at once that every yard of cut C_1 will make s_1 yards of fill wherever it may be deposited, and that the whole of cut C_1 will make C_1s_1 yards of fill whether the material be all dumped in one body or scattered in both fills. On the contrary it will take more or fewer yards in place to make fill F_1 according to the cut from which the material is taken, and so for fill F_2.

After the planning is completed we know where the material for each fill is to come from and can then compute the proper equating-factor, and finally reduce all volumes to volumes in place.

When the engineer has no reliable data to the contrary he will adopt unity as the swell-factor, that is, assume that a yard of cut will make a yard of fill, for the excellent reason that by so doing the computations are reduced to a minimum.

206. Limiting distance (" limit of profitable haul ") when there is no free haul. — The limiting distance — often called " limit of profitable haul" — is discussed in this Section from the standpoint of the contractor and of the owner who does his own grading. Cost, in this section, means cost to the contractor, or to an owner who does his own grading. For a discussion of limiting distance from the standpoint of the owner when the grading is to be done under an overhaul contract see Sec. 207.

The cost of grading, per cubic yard, is composed of (1) cost of excavating and loading; (2) cost of hauling; and (3) cost of dumping. The cost of hauling, only, increases with the distance of hauling.

Referring to Fig. 41 and assuming for the moment that there is no swell of material: We can excavate a cubic yard of material at m, haul it to n, and place it in the fill, at a cost of K cents, say. Or, we can excavate the cubic yard at m and throw it to one side — waste it — and excavate another cubic yard alongside n and place it in the fill at n — borrow a yard at n — at a cost of K' cents, say. Now the useful results are the same in both cases; by each plan a yard of excavation is made at m and a yard of fill is made at n. Assuming that the cost of excavating, loading, and dumping is constant, which of the two is the more economical plan? Evidently when m and n are very near together the first plan is the cheaper; and when m and n are very far apart the second plan is the cheaper. There is, then, some distance between m and n within which the cut-to-fill plan, and beyond which the waste-and-borrow plan, is the more economical. This particular distance we shall call the "limiting distance," and designate by p.

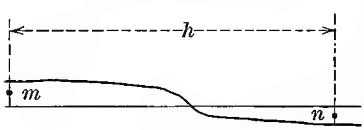


FIG. 41.

We have so far in this section assumed a swell-factor of unity for the material wasted and the material borrowed. In the following derivation of a formula for limiting distance this assumption is omitted.

Let W = total cost of wasting one cubic yard (in place) at m;

B = total cost of borrowing one cubic yard (in place) at n;

K = cost of excavating one cubic yard (in place) at m and dumping it in the fill at n (this does not include the cost of hauling);

T = cost of hauling one cubic yard (in place) 100 ft., that is, cost of haul per station-yard;

s_m = swell-factor for the material at m;

s_b = swell-factor for the material borrowed for the fill at n;

h = mn expressed in stations;

p = limiting distance expressed in stations.

If we haul one cubic yard from m to n we complete 1 c.y. of excavation at m and $1 \times s_m$ c.y. $= s_m$ c.y. of fill at n, at a total cost of $K + Th$.

If, instead of hauling from m to n, we waste the one cubic yard at m we must borrow sufficient material to make the s_m c.y. of fill at n. Since 1 c.y. of borrow makes $1 \times s_b$ $= s_b$ c.y. of fill, to make 1 c.y. of fill requires $\frac{1}{s_b}$ c.y. of borrow; therefore it will require $\frac{s_m}{s_b}$ c.y. of borrow to make s_m c.y. of fill at n. The cost of wasting 1 c.y. at m is W. The cost of the $\frac{s_m}{s_b}$ c.y. of borrow for n, at B per c.y. in place, is $B\frac{s_m}{s_b}$. Therefore the total cost of wasting 1 c.y. at m and providing the corresponding borrow at n is $W + B\frac{s_m}{s_b}$.

To sum up. Whether we haul material from m to n, or waste at m and borrow at n, the useful result accomplished is the same: 1 c.y. of excavation is completed at m and s_m c.y. of fill is made at n. The total cost of 1 c.y. of excavation at m and s_m c.y. of fill at n,

if effected by hauling from cut to fill, $= K + Th$;

if effected by waste and borrow, $= W + B\frac{s_m}{s_b}$.

Evidently it is cheaper to haul from cut to fill instead of wasting and borrowing, so long as the haul distance, h, is such as to make

$$K + Th < W + B\frac{s_m}{s_b}. \tag{41}$$

It is cheaper to waste and borrow than to haul from cut to fill, when the haul distance, h, is such as to make

$$K + Th > W + B\frac{s_m}{s_b}. \tag{42}$$

Both plans are equally economical when the haul distance, h, is such as to make

$$K + Th = W + B\frac{s_m}{s_b}. \tag{43}$$

That value of h which reduces the inequalities of eqs. 41 and 42 to the equality of eq. 43 is the limiting distance which we call p. Thus the limiting distance, when there is no free haul, is

$$p = \frac{\left(W + B\frac{s_m}{s_b} - K\right)}{T}. \tag{44}$$

If $s_m = s_b$, that is, if the swell of the two materials is the same, eq. 44 becomes

$$p = \frac{(W + B - K)}{T}. \tag{45}$$

Looking at eq. 44 we see that the limiting distance increases

with decreasing haul price, T;
with increasing cost of borrowing, B;
with increasing cost of wasting, W;
with decreasing cost of excavating and dumping, K;
with increasing swell-factor, s_m; and
with decreasing swell-factor, s_b.

When estimating the cost of wasting a yard of material the necessary haul distance for waste must be taken into account. Unless there is room for this waste on the right-of-way the land required for the waste dumps must be purchased or the right to waste on private lands must be acquired; and the cost of such land or right, per yard of waste, is a part of W. For a given cost of such land or right, the greater the number of yards to be wasted the less will be the cost per yard arising from cost of land or right. Sometimes the conditions are such that no land in the immediate vicinity of the material to be wasted is available and then the cost of wasting is increased by the longer haul distance, as well as by the cost of the land.

When estimating the cost of a yard of borrow we have to take into account the cost of excavating the material which is available within a minimum distance. If the nearest available material is solid rock we naturally look farther for material which can be excavated at a cost enough smaller than that of excavating solid rock to more than offset the increased cost of hauling. (Solid rock as borrow has some points in its favor: it makes a fill of first quality; and it has a high swell-factor which means that one yard of solid rock will make more fill than one yard of material which is more cheaply excavated.) In some cases the material for borrow is not available on the right-of-way, and it becomes necessary to buy lands for borrow pits or to buy the right to borrow material from private lands. The cost of the land or right, per cubic yard of borrow, is a part of the total cost of a yard of borrow.

It is important to bear in mind that the desirability of a piece of land for borrow or for waste depends somewhat on its elevation with respect to subgrade, on the intervening topography, on the character of the grading plant used, and on the quantity of waste or borrow contemplated. Further, the desirability of a piece of land for borrow will depend on the horizontal and vertical dimensions of the body of material which it is desired to borrow, as well as on the amount of borrow required. If a section of the road involves heavy grading throughout its length so that it is regarded as a steam-shovel job, and the contractor's bid was tendered and accepted on that supposition, it cannot be expected that upon that section other than a steam shovel will be used for borrow; and hence, under the conditions stated, areas of shallow earth cannot be counted on for cheap borrow.

The foregoing principles will now be applied to two examples.

Example 1. — What is the limiting distance under the following conditions?

Cost of excavating, loading, and dumping 1 c.y. of the cut	25 c.
Cost of hauling, per station-yard	1 c.
Total cost of wasting 1 c.y. from the cut	35 c.
Total cost of borrowing 1 c.y. for the fill	30 c.
Swell-factor for the material in the cut	1.10
Swell-factor for the material borrowed	1.00

The limiting distance is

$$p = (0.35 + 0.30 \times 1.10/1.00 - 0.25)/0.01 = 43 \text{ stas.}$$

Example 2. — What is the limiting distance in Example 1 if total cost of wasting is taken at 70 c.; the cost of excavating, loading, and dumping, per cubic yard from the cut, is taken at 65 c.; and the swell-factor of the material of the cut is taken at 1.50?

$$p = (0.70 + 0.30 \times 1.50/1.00 - 0.65)/0.01 = 50 \text{ stas.}$$

207. Limiting distance when there is free haul. — In this Section it is assumed that the grading is done under an over-

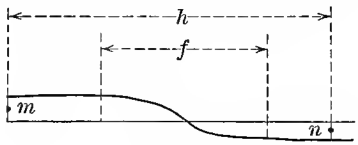


FIG. 42.

haul contract; and the limiting distance is figured from the standpoint of the owner rather than from that of the contractor. Cost, in this Section, means the cost to the owner, not the cost to the contractor. For the limiting distance where there is no free haul see Sec. 206.

Referring to Fig. 42:

Let W = total cost of wasting 1 c.y. (in place) at m;
B = total cost of borrowing 1 c.y. (in place) at n;
K = total cost of excavating 1 c.y. in place at m, hauling it not to exceed the free-haul distance f, and dumping it in the fill at n (this excludes the cost of overhaul);
T = cost of hauling 1 c.y. (in place) 100 ft., that is, the cost of overhaul per station-yard;
s_m = swell-factor for the material excavated at m;
s_b = swell-factor for the material borrowed for the fill at n;
h = mn expressed in stations;
f = free-haul distance expressed in stations;
p = limiting distance expressed in stations.

The cost of excavating 1 c.y. at m and hauling it to and dumping it in the fill at n is $K + T(h - f)$. On the other hand, the cost of wasting 1 c.y. at m and borrowing sufficient material to make s_m c.y. of fill at n is $W + B\frac{s_m}{s_b}$. The limiting distance, when there is free haul, is that value of h which makes the two costs, just given, equal. Therefore

$$p = \frac{W + B\frac{s_m}{s_b} - K}{T} + f. \qquad (46)$$

Example 1. — What is the limiting distance under the following conditions?

Free-haul distance	300 ft.
Contract price per yard, for material moved less than free-haul distance { from the cut	50 c.
{ from the borrow pit	20 c.
Overhaul price, per cubic yard in place moved 100 ft.	1 c.
Swell-factor for the material in the cut	1.20
Swell-factor for the material in the borrow pit	.90

Material is wasted and borrowed within the free-haul distance.

The limiting distance is

$$p = (0.50 + 0.20 \times 1.20/0.90 - 0.50)/0.01 + 3 = 29.67 \text{ stas.}$$

Example 2. — What is the limiting distance in Example 1 if the material from the cut cannot be wasted with a haul distance shorter than 800 ft.? Under the changed conditions $W = K + T \frac{(800 - 300)}{100} = K + 5\,T = 0.50 + 5\,(0.01) = 0.55$;

and $p = (0.55 + 0.20 \times 1.20/0.90 - 0.50)/0.01 + 3 = 34.67$ stas.

208. Principle of economic distribution. — Other things being equal, that is the most economical distribution of material along the profile, which result in minimum total haul (sta-yds.). The application of this principle is made by means of the mass curve to some special cases in the following chapters. In Chapter IX it is throughout assumed that the mass curve is plotted: (1) from actual cut-volumes and actual fill-volumes, the cuts having each a swell-factor of 1; or (2) from actual cut-volumes and equated fill-volumes. In Chapter X it is assumed throughout that the mass curve is plotted from actual fill-volumes and equated (i.e., swelled) cut-volumes.

NOTE. — Under some conditions of grading it is not on the whole economical to adhere strictly to a plan of distribution which involves minimum haul. This point is discussed in Note 3, Sec. 213, which see.

CHAPTER IX

ECONOMIC BALANCING LINES FOR MASS CURVES PLOTTED FROM CUT-VOLUMES AND EQUATED FILL-VOLUMES, OR FROM CUT-VOLUMES AND FILL-VOLUMES WHEN SWELL-FACTOR IS UNITY

This chapter discusses the application of the principle of economic distribution to each of several typical forms of mass curve, progressing from the simple to the complex. All the mass curves of this chapter are assumed to be plotted from pay yards, that is, from yards in place. All the mass curves of the following chapter are assumed to be plotted from yards of swelled material. Otherwise the two chapters are similar. Figs. 50–60a face page 146.

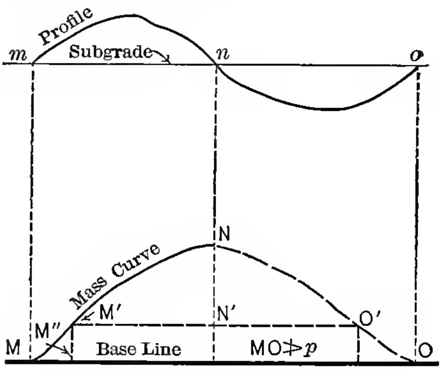
209. **Note.** — It is important to bear in mind that, because every mass curve in this chapter (1) represents grading in which there is neither swell nor shrinkage, or (2) is plotted from actual cut-volumes and equated fill-volumes, the balancing line in every case must be horizontal, and haul = haul-area multiplied by $\frac{YS}{100}$ (see Sec. 14).

For the economic balancing line for mass curve, plotted from fill-volumes and equated cut-volumes, see Chapter X.

It is, of course, necessary to use estimated volumes of cut and of fill in the work of planning distribution.

210. **Economic balancing line for the simple loop.** — Between two adjacent zero ordinates of the mass curve the distance between which is not greater than the limiting distance, the base line is the economic balancing line, provided no adjacent loop lying on the same side of the base line is nearer than the limiting distance.

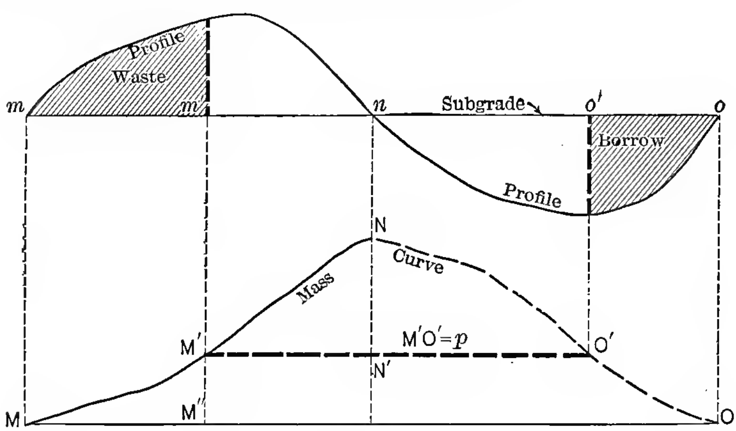
Let it be assumed that the profile *mo*, Fig. 43, represents a self-contained stretch of earthwork. Let the cut *mn* be assumed to contain just sufficient material to make the fill *no*. This is indicated by the mass curve which rises from M to N and then descends returning to the base line at O. Let MO be not greater



MO is Economic Balancing Line

FIG. 43.

than the limiting distance (Secs. 206 and 207). In this case MO, the base line, is the economic balancing line.



M'O' is the Economic Balancing Line

FIG. 43a.

Area MNO is the measure of the haul (sta-yds.) involved in the grading, that is, the haul is proportional to this area (Sec. 14).

If we raise the balancing line somewhat, say to the position $M'N'O'$, we reduce the haul-area by $MM'O'O$; but this implies wasting $M'M''$ c.y. between M and M', and borrowing the same amount between O' and O. In this case, then, it is possible to decrease the haul corresponding to the balancing line MO only at the cost of introducing waste and borrow. Since MO is less than the limiting distance, the introduction of waste and borrow is not economical (Secs. 206 and 207). Hence the balancing line MO is the economic balancing line.

In the cases in which MO is greater than the limiting distance the economic balancing line is that horizontal line which cuts the curve in two points between which the distance is equal to the limiting distance. Thus in Fig. 43a, $M'O'$ ($= p$) is the economic balancing line.

211. Economic balancing line for single corrugated loop. — Let us consider the haul in Fig. 44 in which the mass curve leaves

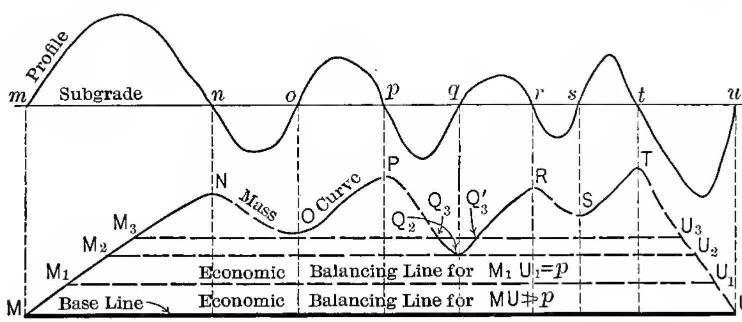


FIG. 44.

the base line at M and returns to it at U, between which points there are several cuts and fills. There are three cases to be considered.

(1) Assume MU to be not greater than the limiting distance, p. In Fig. 44, as in Fig. 43, the base line is the economic balancing line, because the area between the base line MU and the mass curve $MNO \ldots U$, which area is the measure of the haul

which results from using the base line as a balancing line, can be diminished only by raising the balancing line; and raising the balancing line introduces waste and borrow which under the conditions named is not economical.

(2) Assume that MU is greater than p, the limiting distance. Then the economic balancing line is M_1U_1 so placed that $M_1U_1 = p$, provided M_1U_1 does not touch the curve at any point between M_1 and U_1.

(3) Draw the horizontal $M_2Q_2U_2$ tangent at Q_2 to the lowest sag in the mass curve. Draw M_3U_3 to cut the mass curve at two points only between M_3 and U_3. The two intermediate points are Q_3 and Q_3'.

Assume that M_2U_2 is greater than p. This case falls under Sec. 210 or Sec. 212.

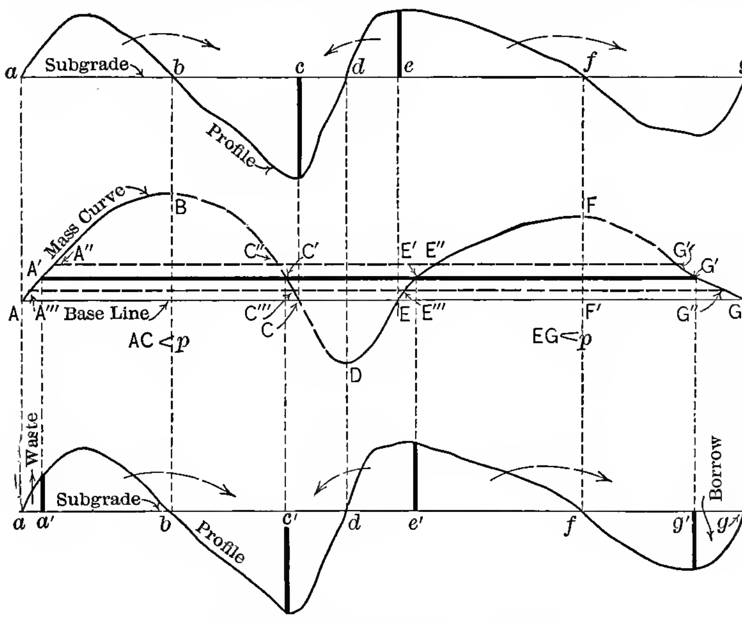
212. Economic balancing line for two long loops separated by a short loop. — In Secs. 210 and 211 it has been shown that the base line is, under limiting conditions, the economic balancing line for a single isolated loop, whether simple or corrugated.

If the mass curve in question contains two closely adjacent loops on the same side of the base line, as in Fig. 45, the work of finding the economic balancing line is more complex.

It is assumed that AC and EG, Fig. 45, are each less than p, the limiting distance. The economic balancing line for loop ABC, when that loop is considered by itself, is the base line AC. So, too, the base line EG is the economic balancing line for loop EFG when that loop is considered apart from the rest of the mass curve.

Now let us consider at one time the whole mass curve between A and G. Adopting the balancing line AG results in the distribution indicated by the arrows in Fig. 45a: cut ab makes fill bc; cut de makes fill cd; cut ef makes fill fg. The total haul is represented by the sum of the areas ABC, CDE, and EFG.

Next, let us shift the balancing line to a higher position, $A'G'$. In thus raising the balancing line we effect three changes: (1) We decrease the total haul-area by the sum of the two strips $AA'C'C$ and $EE'G'G$, and this decrease is an advantage; (2) we increase the total haul-area by the strip $CC'E'E$; and this is a disadvantage; and (3) we introduce waste between A and A', and borrow between G and G'; and this is a disadvantage. The distribution resulting from the balancing line $A'G'$ is shown by



If $A'C' + E'G' - C'E' = p$, $A'G'$ is the economic balancing line

FIGS. 45 (middle), 45a (upper), 45b (lower).

arrows in Fig. 45b. Whether or not it is economical to raise the balancing line above the base line depends upon the relation between the advantage of decreasing the haul (sta-yds.) and the corresponding disadvantage of introducing waste and borrow. If $A'G'$ is so drawn as to make

$$A'C' + E'G' = C'E' + p, \tag{47}$$

or

$$A'C' + E'G' - C'E' = p, \tag{48}$$

where p is the limiting distance; then $A'G'$ is the economic balancing line. This may be shown as follows:

If we drop the balancing line $A'G'$ (so drawn as to produce the equality above written), the yard at A' goes to C', giving a haul $= A'C'$ sta-yds.; the yard at E' goes to G', giving a haul $= E'G'$ sta-yds.; the total haul for the two yards being $A'C' + E'G'$ sta-yds.

If we raise the balancing line above $A'G'$, the yard at E' goes to C', giving a haul $= E'C'$ sta-yds. The yard at A' is wasted, and the yard at G' must be borrowed, thus giving a haul of $E'C'$ plus a yard of waste plus a yard of borrow, and this is equivalent to haul $E'C' + p$, since one yard of waste plus one yard of borrow is equivalent to one yard hauled a distance p (Secs. 206 and 207). Since by assumption $A'C' + E'G' = C'E' + p$ it is plain that it is a matter of indifference whether we haul the yard at A' to C' and the yard at E' to G', or haul the yard at E' to C', and waste the yard at A' and borrow a yard for G'.

Now if we raise the balancing line one cubic yard (to scale) above $A'G'$ to the position $A''G''$,

$$A''C'' + E''G'' < C''E'' + p, \tag{49}$$

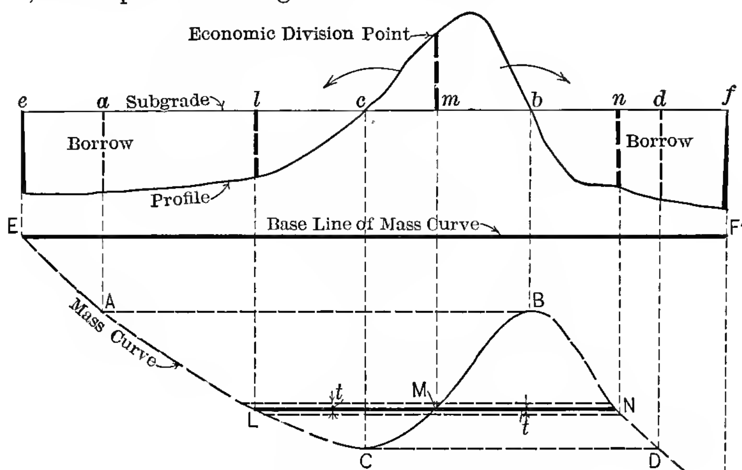
which means that it is cheaper to haul the yard at A'' to C'' and haul the yard at E'' to G'', than to haul the yard at E'' to C'', waste the yard at A'', and borrow the yard for G''. On the other hand, if we lower the balancing line by one cubic yard (to scale) from $A'G'$ to $A'''G'''$,

$$A'''C''' + E'''G''' > C'''E''' + p, \tag{50}$$

which means that it is cheaper to haul the yard at E''' to C''', waste the yard at A''', and borrow the yard for G''', than to haul the yard at A''' to C''', and haul the yard at E''' to G'''.

213. Economic balancing line for a mass curve of one sag and one hump. — Fig. 46 shows a self-contained section of earthwork. The mass curve shows that the cut is not sufficient to complete either of the two fills. It is assumed that the necessary borrow can be obtained as advantageously at one point of the profile as at another.

The question is: Shall the cut all be hauled to one fill? If so, to which fill? If not, what part of the cut shall be hauled to the left; what part to the right?



If LM=MN, LN is the Economic Balancing Line

FIG. 46.

Draw the balancing line *AB*. With this balancing line the whole cut is hauled to the left, completing the fill to *a*. The fill to the left of *a* as well as the fill to the right of *b* must be made of borrow. Neglecting the borrow, for that will be the same whatever disposition is made of the cut, the haul resulting from the balancing line *AB* is proportional to the area *ACB*.

Next, try the balancing line *CD*. This results in hauling all the cut to the right, completing the fill to *d*. The required borrow is the same with this as with the first balancing line. The area *CDB* is the measure of the haul involved in moving all the cut to the right.

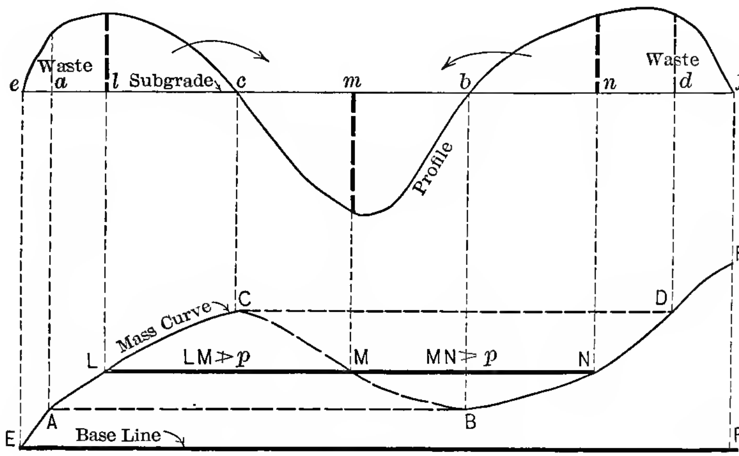
If the area *CBD* is less than the area *ACB* the cut should be taken to the right rather than to the left. However, on further consideration we shall find that neither *AB* nor *CD* is the economic balancing line for this mass curve.

Draw by trial the balancing line *LMN* so as to make *LM* = *MN*. *LMN* is the economic balancing line. Using this balancing line, the cut to the left of *m* makes the fill to *l*; and the resulting haul is measured by the area *LCM*. The cut to the right of *m* makes the fill to *n*; and the resulting haul is measured by the area *MBN*. The total haul with this balancing line is thus measured by total area, *LCM* + *MBN*. We now go on to show that this total area is a minimum.

If we raise the balancing line *LMN* slightly — say a distance *t* — the right-hand haul-area is decreased by the strip above *MN*; but the left-hand haul-area is at the same time increased by the longer strip above *LM*; and the net result of raising the balancing line the slight distance, *t*, is to increase the total haul-area. Evidently by raising the balancing line step by step — a distance *t* each time — the strip added at each step to the left-hand haul-area is longer than the corresponding strip taken away from the right-hand area. This means that any position of the balancing line above *LMN* involves more haul-area, and therefore more haul, than the balancing line *LMN*.

Again, try the effect on haul-area of lowering the balancing line from the position *LMN*. By dropping *LMN* a small distance, *t*, the haul-area on the left is decreased by the strip lying below *LM*; but at the same time the longer strip below *MN* is added to the haul-area on the right. Hence the net result of lowering the balancing line by the small distance *t* is an increase in the total haul-area. If we go on lowering the balancing line step by step, each successive step increases the right-hand haul-area more than it decreases the left-hand haul-area. Thus it appears that any position of the balancing line below *LMN* involves more haul-area, and therefore more haul, than *LMN*.

To sum up. The balancing line *LMN* is so drawn (by trial) that the two segments, *LM* and *MN*, are equal. Any balancing line, above or below *LMN*, involves more haul-area, and therefore more haul, than *LMN*. Therefore *LMN* is the economic balancing line.



If LM=MN, LN is the Economic Balancing Line

FIG. 46a.

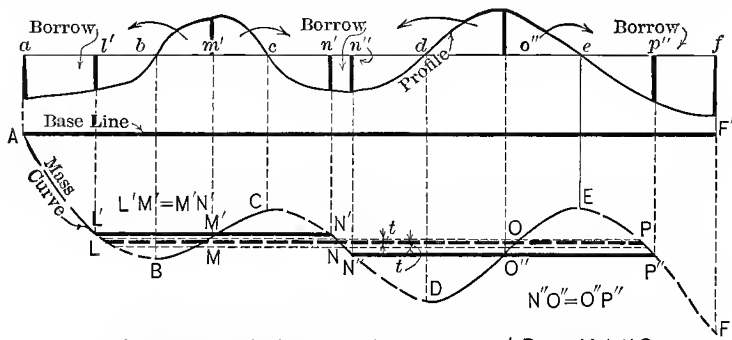
NOTE 1. — Another way of stating the proposition above is: for the conditions shown in Fig. 46, the position of the economic division point in the cut is that which makes the extreme haul-distances equal. Thus the proposition is stated by Mr. George H. Tinker who gave an algebraic proof of it in *Engineering News*, 1901, vol. 45, p. 82.

NOTE 2. — The same proposition applies, of course, in the case of a profile which shows a fill lying between two cuts either one of which is sufficient to make the complete fill. Thus in Fig. 46a, *LMN*, which is so drawn as to make *LM* = *MN*, is the economic balancing line.

NOTE 3. — Fig. 46. The division point *m* of the cut has been so located as to give the least total haul. If the material in the cut is not divided at *m* there will be unnecessary haul. Now, under some conditions of grading it is not economical, on the whole, to eliminate all such unnecessary haul. For example, if the material is moved with carts, and hauling against grade is more expensive than with the grade, by reason of the steepness of the grade or the presence of water in the up-grade cut, it will be unwise to adhere to the plan of dividing the cut precisely at *m*. Again, if the cut is taken out by means of a steam-shovel plant the economical working of that plant will probably require wide departure from the plan of distribution which involves the least haul — perhaps even to the extent of hauling all of the cut in one direction if the topography at one end of the cut permits a low grade for the hauling track while the topography at the other end requires an excessive grade.

The fact that in practice it is often wise to depart more or less from the plan which involves the least haul is not an argument against making such plan. It is sound engineering to make such plan of distribution and depart from it only in so far as good reasons for so doing present themselves.

214. Economic continuous balancing line for a mass curve with two sags and two humps. — Fig. 47 shows a profile of a self-



The Economic Continuous Balancing Line is LP if LM + NO = MN + OP; but the Broken Balancing Line L'N', N''P'' is the Economic Balancing Line in this Case

FIG. 47.

contained stretch of earthwork; that is, no material is moved past *a* or *f*. The mass curve, drawn below the profile, shows that there is more fill than cut. Material to the amount represented by the ordinate FF' must be borrowed. Let it be assumed that borrow is as readily got at one part of the profile as at another.

The line LP, drawn by trial so as to make

$$LM + NO = MN + OP, \tag{51}$$

is the economic continuous balancing line for this mass curve, because it gives the least total haul-area, as shown thus: if we raise the balancing line LP a small distance t, we add two strips of haul-area above LM and NO, which strips have a greater total length than the total length of the two strips below MN and OP which are taken away from the haul-areas. The net result of raising the balancing line above the position LP is, therefore, to increase the total haul-area. In like manner it may be shown that to drop the balancing line below the position LP is to increase the total haul-area. Hence LP is the economic continuous balancing line.

The same principle will enable us to place the economic continuous balancing line across any equal number of humps and sags. This principle is given on page 361 of Proceedings, American Railway Engineering Association, vol. 7 (1906), and its first use in America is attributed to Mr. F. Reineker.

In Sec. 215 it will be shown that the economic *continuous* balancing line may or may not be *the* economic balancing line.

215. Continuous balancing line vs. broken balancing line. — We have shown that LP is the economic *continuous* balancing line for the profile of Fig. 47. Now we shall show that for this case there is a broken balancing line which is *the* economic balancing line.

Let LP be broken at N. Raise LN to the position $L'N'$, such that

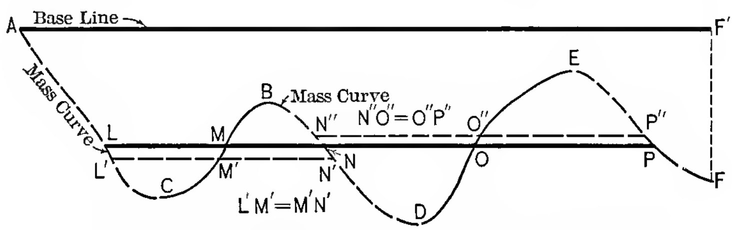
$$L'M' = M'N'. \tag{52}$$

Lower NP to the position $N''P''$, such that

$$N''O'' = O''P''. \tag{53}$$

It is apparent that the broken balancing line $L'N'$, $N''P''$ is *the* economic balancing line for this case. The only effect which this breaking of the continuous balancing line has on the borrow is to shift borrow $L'L$ to $N'N$, and borrow PP'' to NN''.

It may appear at first glance that the economic continuous balancing line can always be broken to advantage; but this is not so. Fig. 48 shows a mass curve for which LP, so drawn as to make $LM + NO = MN + OP$, is *the* economic balancing line. If we break LP at N, and shift each part, regardless of the other, to obtain a minimum total haul-area for each part, we find that LN must be lowered to the position $L'N'$ for which $L'M' = M'N'$; and NP must be raised to the position $N''P''$ for which $N''O'' = O''P''$. But this shifting of the two parts



The Economic Continuous Balancing Line LP is the Economic Balancing Line in this Case

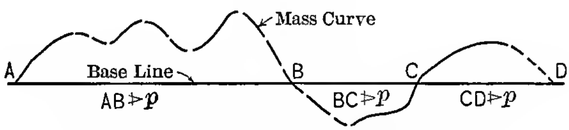
FIG. 48.

causes them to overlap between N'' and N'; and such overlapping is inadmissible. In the case shown in Fig. 48 the continuous economic balancing line is therefore *the* economic balancing line. It may be stated as a general rule that the continuous economic balancing line is *the* economic balancing line except when it can be broken into parts and each part, considered by itself, shifted to the position of least haul without an overlapping of the parts.

216. When the base line is the economic balancing line. — In Fig. 49, which represents a mass curve constructed on the base line AD, let it be assumed that the length of each of the segments into which the base line is cut by the mass curve is not greater than the limiting distance, p (Secs. 206 and 207). The mass curve shows that the cuts and fills balance between the two points A and D. Let it be assumed that no material is to be moved past A or D. Let it be assumed, further, that

$$AB + CD - BC < p. \qquad (54)$$

Under the foregoing conditions the economic balancing line is the base line AD. This is true because to raise or lower the balancing line AD or any part thereof is to introduce waste and borrow; and under the given conditions waste and borrow are not economical.



The Base Line is the Economic Balancing Line

FIG. 49.

217. Examples of economic balancing lines. — It is assumed that the conditions are such that no material will be moved past either end of the profile in any of the figures (folding plate, page 146) to which reference is made in this section. Each profile is accompanied by its mass curve.

On each mass curve the economic balancing line is represented by a full, heavy line and is designated by two or more of the letters L, M, N, L', M', N'. . . .

The economic distribution of material, corresponding to the economic balancing line, is indicated by arrows on the profile.

The figures are given in pairs; the first figure of each pair shows the mass curve above the base line; the second shows the mass curve below the base line. The two figures of each pair present the same problem as far as determining the economic balancing line.

In every case p = limiting distance (Secs. 206 and 207).

Fig. 50. — The profile shows a cut only. There is no fill into which the material of the cut may be dumped; hence the entire cut must be wasted. This condition is clearly reflected in the mass curve, which, rising from start to finish, precludes a balancing line. Fig. 50a shows a fill the material for which must be borrowed.

Fig. 51. — Here we have a cut and a fill. The cut *ab* is more than sufficient to make the fill *bm*. Hence there is waste. This condition is shown by the fact that the right extremity of the mass curve lies above the base line. Draw a horizontal through M to meet the mass curve at L. If LM is not greater than p, the limiting distance, LM is the economic balancing line. Assuming that $LM \ngtr p$, draw the vertical Ll. The portion *lb* of the cut just makes the fill *bm*. The portion *al* of the cut is wasted. This is the economical distribution of the material, and is indicated by arrows on the profile. In Fig. 51a we have the foregoing conditions reversed: the fill *ab* is larger than the cut *bm*, so that the fill *al* is borrowed, and fill *lb* takes the entire cut, *bm*.

Fig. 52. — The right extremity, C, of the mass curve lies at a distance CC' above the base line, which means that waste to the amount of CC' (to scale) cannot be avoided. We drew through C the horizontal CA' meeting the left slope of the mass curve at A'. We scaled the distance $A'C$ and found it to be greater than p, the limiting distance. Therefore $A'C$ cannot be the economic balancing line. Next we raised the balancing line to the position LM such that $LM = p$. LM is the economic balancing line. Hence the economic distribution: cut *lb* makes fill *bm*; cut *al* is wasted; and fill *mc* is borrowed. In Fig. 52a we have a similar case, except that the fill is larger than the cut. Hence borrowing to the amount CC' (to scale) cannot be avoided. $A'C$ is greater than p; therefore $A'C$ is not the economic balancing line. The economic balancing line in this figure is LM so drawn as to make $LM = p$. The arrows show the economic distribution.

Fig. 53. — Here the cut balances the fill. That is to say, the material of the cut is just sufficient to make the fill. This condition is shown by the mass curve which begins and ends on the base line. It is assumed that the base line intercept, LM, is not greater than p, the limiting distance. In this case the economic

balancing line is LM coincident with the base line. Fig. 53a tells a similar story.

Fig. 54. — Here, too, the cut balances the fill: the mass curve starts from and returns to the base line. The base line intercept, AC, is assumed to be greater than p, the limiting distance. Consequently AC is not the economic balancing line. The economic balancing line is LM so drawn that $LM = p$. These remarks apply also to Fig. 54a.

Fig. 55. — The mass curve starts on the base line and ends below the base line. The fill exceeds the cut by CC' (to scale) cubic yards. Hence borrow, to the amount CC', is unavoidable. Assume the base line intercept to be not greater than p, the limiting distance. The base line is the economic balancing line. This case is then just like that of Fig. 53 except for the need of borrowing. If the base line intercept were greater than p the economic balancing line would be determined as in Fig. 54. Fig. 55a shows waste instead of borrow, but presents the same problem as Fig. 55.

Fig. 56. — The profile shows a fill between two cuts. The mass curve, beginning on the base line and ending at N above the base line, shows that there is unavoidable waste to the extent of NN' (to scale) cubic yards. The base line cannot be the balancing line in this case because it is touched by the mass curve at only one point. The lowest balancing line which can be drawn is CA', a horizontal through C. $A'C$ is not the economic balancing line. Draw a horizontal through N cutting the mass curve at L and M. Assuming that the intercept LM is not greater than p, the limiting distance, and that MN is not greater than LM, the economic balancing line is LN. Fig. 56a shows a cut between two fills, but the problem presented is the same as that in Fig. 56.

Fig. 57. — The conditions here differ from the conditions of Fig. 56 in this: On drawing the horizontal through N, Fig. 57, we find the intercept $A'M$ is greater than p, the limiting distance.

Here economy requires that we break the balancing line $A'N'$ at M, and raise the left-hand portion to the position $L'M'$ such that $L'M' = p$. $L'M'$, MN is the economic balancing line. The arrows indicate the economic distribution. The same remarks apply to Fig. 57a.

Fig. 58. — Here, as in Figs. 56 and 57, there is a fill lying between two cuts. Draw a horizontal through D, the right-hand end of the mass curve, cutting the mass curve at B' and B''. Assume that $B''D$ is greater than $B'B''$. $B'B''D$ is not the economic balancing line. Draw the horizontal LM so as to make the two intercepts, LM and MN, equal. Assuming that neither LM nor MN is greater than p, the limiting distance, LMN is the economic balancing line. Fig. 58a shows a cut lying between two fills but the problem of determining the economic balancing line is the same as in Fig. 58.

Fig. 59. — The conditions here are like those of Fig. 58, except that here we find on drawing a horizontal LN so as to make $LM = MN$, that both LM and MN are greater than p, the limiting distance. Hence, LMN is here not the economic balancing line. Economy requires us to break the balancing line LMN at M; raise the left portion to the position $L''M''$ such that $L''M'' = p$; and lower the right portion to the position $M'N'$ such that $M'N' = p$. The broken balancing line $L''M''$, $M'N'$ is the economic balancing line. Fig. 59a presents the same problem.

Fig. 60. — Here again we have a fill lying between two cuts. The fill in this case, however, is greater than the combined cuts, and this means unavoidable borrow. Draw through N', the right end of the mass curve, a horizontal cutting the curve at M'. Assuming that neither LM, the base line intercept, nor $M'N'$ is greater than p, the limiting distance, the broken balancing line LM, $M'N'$ is the economic balancing line. Fig. 60a shows a cut lying between two fills, the cut being more than sufficient to make the fills, so that there is unavoidable waste. The problem of Fig. 60a is the same as that of Fig. 60.

218. Practical use of mass curve in planning distribution. — Mass curves in practice are commonly much longer than any of those which appear in this book. It is believed, however, that any mass curve, no matter how long it may be, can be readily divided into segments each of which presents a problem of distribution separate from the problems of the adjacent segments; and that each individual problem thus segregated will fall under some one of the special cases which are presented in the foregoing and following Sections.

When the mass curve is used for planning distribution, it must be revised or reconstructed, in whole or in part, as a necessary preliminary to each revision of the plan of distribution, which may be made necessary by discovering, from grading completed or in progress, a material difference between some of the volumes and swell-factors used in plotting the mass curve, and the corresponding actual volumes and swell-factors.

The engineer can plan distribution, under the conditions of this chapter, much more rapidly and satisfactorily by the use of the mass curve than otherwise, provided he be familiar with such use. Anyone who can plot profiles can be taught quickly to construct mass curves; hence the engineer can have a mass curve prepared for his use at any time, at little expense. It remains for each engineer to discover by experience whether or not his use of the mass curve is in general desirable, and to decide whether or not to use the mass curve in any given case of making or revising a plan of distribution. See Sec. 226.

CHAPTER X

ECONOMIC BALANCING LINE FOR MASS CURVE PLOTTED FROM FILL-VOLUMES AND EQUATED CUT-VOLUMES

This chapter discusses the application of the principle of economic distribution to each of several typical forms of mass curve, progressing from the simple to the complex. The mass curves here represent cubic yards of swelled material; while the mass curves of the preceding chapter represent cubic yards in place.

219. Note. — Every mass curve in this chapter is plotted from the estimated fill-volumes and equated estimated cut-volumes. This means that each station-volume of cut was multiplied by its estimated swell-factor, and the resulting swelled volumes were combined with the estimated fill-volumes to obtain the ordinates to the mass curve.

For economic balancing lines for mass curves plotted from cut-volumes and equated fill-volumes, and for mass curves plotted from fill-volumes and cut-volumes of unity swell see Chapter IX.

220. Economic balancing line for simple loop. — The base line, Fig. 62, is the economic balancing line provided AC is not greater than p, the limiting distance; and the

$$\text{total haul} = \left(\frac{(\text{area } ABC)}{s}\right)\frac{YS}{100}, \tag{55}$$

where s = swell-factor for cut ab; Y = cubic yards per inch of ordinate; and S = feet per horizontal inch of the mass curve.

If AC is greater than p, the economic balancing line is $A'C'$ so placed that $A'C' = p$; and the total haul $= \left(\frac{1}{s}\right)$ (area $A'BC'$) $\frac{YS}{100}$ sta-yds; but in this case AA' is wasted and $C'C$ is borrowed.

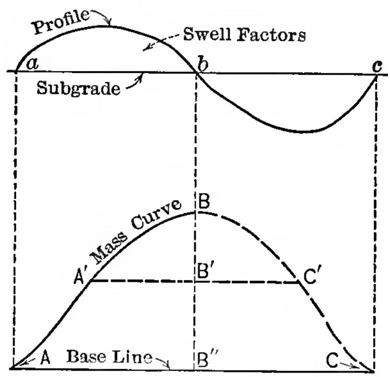


FIG. 62.

221. Economic balancing line for corrugated loop.—Referring to Fig. 63: Let s_1 and s_2 be the swell-factors of cuts C_1 and C_2 respectively. AE is the base line of the mass curve. Draw the horizontal A_2E_2 to touch the sag at C. Draw the horizontal

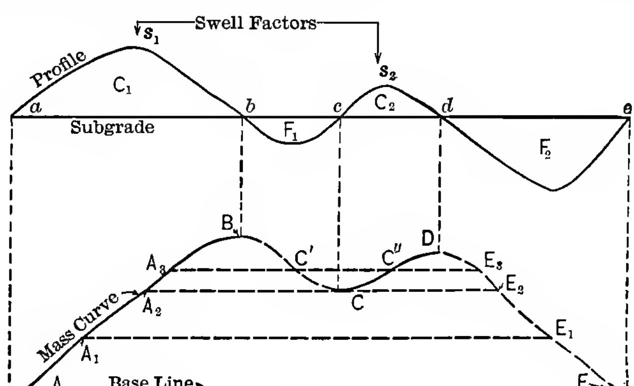


FIG. 63.

A_1E_1 between A_2E_2 and the base line. Draw A_3E_3 above A_2E_2 to cut the sag at C' and C''. Now if A_2E_2 is less than p, the limiting distance, the problem of determining the economic balancing line is the same as if the loop were simple (Sec. 220).

But if A_2E_2 is greater than p, the problem of determining the economic balancing line comes under Sec. 222, which see.

222. Economic balancing line for two long loops separated by a short loop. — Fig. 64. At first glance the base line seems

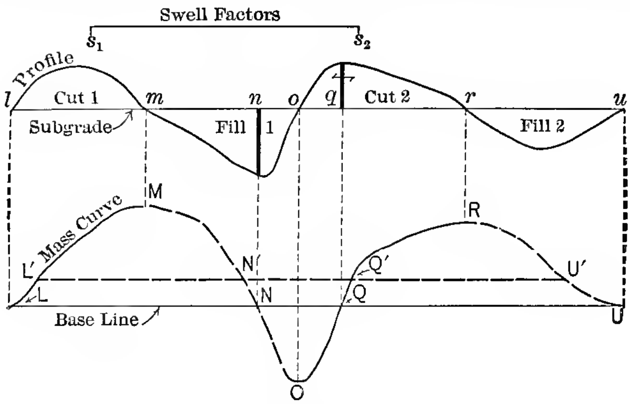


FIG. 64.

to be the economic balancing line, provided neither LN nor QU is greater than p, the limiting distance.

Let W = total cost of wasting 1 c.y. in place at l, less K_1;

B = total cost of borrowing 1 c.y. in place at u;

K_1 = cost of excavating and dumping 1 c.y. (in place) of the material of cut 1;

K_2 = cost of excavating and dumping 1 c.y. (in place) of the material of cut 2;

s_1 = swell-factor of cut 1;

s_2 = swell-factor of cut 2;

s_b = swell-factor of material for borrow at u;

T = cost per station-yard of haul = cost of hauling 1 c.y. (in place) 100 ft.

With the base line as balancing line the distribution is as follows:

material of cut lm makes fill mn;
material of cut oq makes fill no;
material of cut qr makes fill ru.

One yard of fill at n is made by $\frac{1}{s_1}$ c.y. of material from l, requiring haul $= \frac{LN}{s_1}$. One yard of fill at u is made by $\frac{1}{s_2}$ c.y. of material from q, requiring a haul $= \frac{QU}{s_2}$. To make the two yards of fill requires a total haul $= \frac{LN}{s_1} + \frac{QU}{s_2}$; and the cost of making the two yards of fill and the resulting excavation is

$$x = \frac{K_1}{s_1} + \frac{K_2}{s_2} + \left(\frac{LN}{s_1} + \frac{QU}{s_2}\right)T. \qquad (56)$$

The alternative is to make the yard of fill at n with $\frac{1}{s_2}$ c.y. of material from q; requiring haul $= \frac{QN}{s_2}$ and costing $\frac{K_2}{s_2} + \left(\frac{QN}{s_2}\right)T$; make the yard of fill at u with $\frac{1}{s_b}$ c.y. of borrow, at a cost of $\frac{B}{s_b}$; and waste the $\frac{1}{s_1}$ c.y. at l at a cost of $\frac{K_1}{s_1} + \frac{W}{s_1}$. Total cost of alternative is

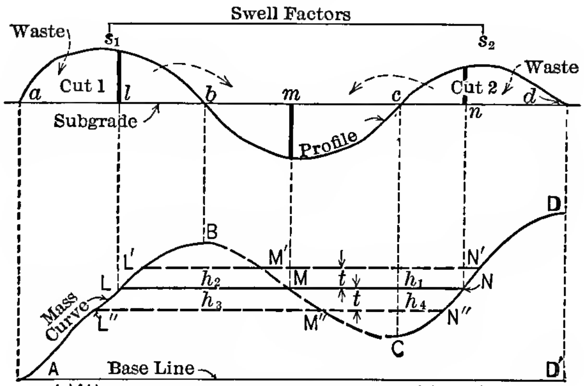
$$y = \frac{K_2}{s_2} + \left(\frac{QN}{s_2}\right)T + \frac{B}{s_b} + \frac{K_1}{s_1} + \frac{W}{s_1}. \qquad (57)$$

The two plans of distribution are equally economical when $x = y$. The two long hauls are economical when $x < y$. The plan of wasting and borrowing is economical when $x > y$. If the latter is the case, the balancing line should be raised to the position $L'N'Q'U'$ such that $x = y$; and this position is found by trial. All the haul-distances must be expressed in stations in the foregoing equations.

223. Economic balancing line for mass curve with one sag and one hump. — Fig. 65. The profile shows a fill lying between two cuts. The mass curve shows that there is unavoidable waste to the amount of DD' (to scale) cubic yards.

Let s_1 = swell-factor of cut 1;
s_2 = swell-factor of cut 2;
Y = cubic yards represented by one inch of ordinate;
S = feet represented by one horizontal inch of mass curve.

Remember that the ordinates of the mass curve are made up from fill-volumes and equated cut-volumes.



LMN is the Economic Balancing Line if LM=MN$\div p$
For meaning of h_1, h_2, etc., see § 223

FIG. 65.

Draw the horizontal LMN so as to make

$$\frac{LM}{s_1} = \frac{MN}{s_2}. \tag{58}$$

LMN is the economic balancing line. This may be shown as follows:

If we raise the balancing line LMN a small distance t to the position $L'M'N'$, we add to the area MCN a strip $MM'N'N$ of which the length is $\frac{1}{2}$ $(MN + M'N') = MN + h_1$, say; and this means an increase of $(MN + h_1)\left(\frac{t}{s_2}\right)\frac{YS}{100}$ sta-yds. of haul. (Secs. 14 and 15.) But at the same time we take away from the haul-area LBM the strip $LL'M'M$ of which the length is $\frac{1}{2}(LM + L'M') = LM - h_2$, say; and this means a decrease of $(LM - h_2)\left(\frac{t}{s_1}\right)\frac{YS}{100}$ sta-yds. of haul. If we let H' = net increase in haul which results from raising the balancing line from the position LMN to the position $L'M'N'$, we have

$$H' = (MN + h_1)\left(\frac{t}{s_2}\right)\frac{YS}{100} - (LM - h_2)\left(\frac{t}{s_1}\right)\frac{YS}{100}. \qquad (59)$$

Now
$$\frac{MN}{s_2} = \frac{LM}{s_1};$$

therefore
$$H' = \left(\frac{h_1}{s_2} + \frac{h_2}{s_1}\right)\frac{tYS}{100}. \qquad (60)$$

Thus we find that this increase, H', in the total haul, due to raising the balancing line the small distance t, is positive. Therefore it is not economical to raise the balancing line above the position LMN.

Again: If we lower the balancing line a small distance t, to the position $L''M''N''$, we add to the haul-area LBM the strip $LL''M''M$, the length of which is $\frac{1}{2}(LM + L''M'') = LM + h_3$, say; and this increases the haul by $(LM + h_3)\left(\frac{t}{s_1}\right)\frac{YS}{100}$. And at the same time we take away from the haul-area MCN the strip $MM''N''N$, the length of which is $\frac{1}{2}(MN + M''N'') = MN - h_4$, say, thus decreasing the total haul-area by $(MN - h_4)\left(\frac{t}{s_2}\right)\frac{YS}{100}$ sta-yds. Let H'' = net increase in haul resulting from this lowering of the balancing line. Then

$$H'' = (LM + h_3)\left(\frac{t}{s_1}\right)\frac{YS}{100} - (MN - h_4)\left(\frac{t}{s_2}\right)\frac{YS}{100}$$

$$= \left(\frac{h_3}{s_1} + \frac{h_4}{s_2}\right)\frac{tYS}{100} \text{ sta-yds.}, \qquad (61)$$

since $\frac{LM}{s_1} = \frac{MN}{s_2}$. H'' is a positive quantity. Therefore it is not economical to lower the balancing line below the position LMN.

We see that either to raise or to drop the balancing line from the position LMN is to increase the total haul. Therefore LMN, the balancing line of minimum haul, is the economic balancing line.

224. Economic balancing line for a mass curve with two sags and two humps. — Fig. 66. The profile shows from left to right a cut, a fill, a cut, a fill, and a cut. The mass curve shows that the material of the cuts exceeds the requirement of the fills by FF' (to scale) cubic yards.

Draw the horizontal LQ so as to make

$$\frac{LM}{s_1} + \frac{NO}{s_2} = \frac{MN}{s_2} + \frac{OQ}{s_3}. \tag{62}$$

$LMNOQ$ is the economic continuous balancing line, as we shall see.

Let s_1 = swell-factor of cut 1;
s_2 = “ “ “ “ 2;
s_3 = “ “ “ “ 3;
h_1 = decrement in LM due to raising LQ a small distance t.
h_2 = increment “ MN “ “ “ “ “ “
h_3 = decrement “ NO “ “ “ “ “ “
h_4 = increment “ OQ “ “ “ “ “ “
h_5 = “ “ LM “ “dropping “ “ “
h_6 = decrement “ MN “ “ “ “ “ “
h_7 = increment “ NO “ “ “ “ “ “
h_8 = decrement “ OQ “ “ “ “ “ “
Y = cubic yards represented by one inch of ordinate;
S = feet represented by one horizontal inch of mass curve.

If we raise the balancing line a small distance, t, above the position LQ, the net increase in haul is

$$H' = \left[\left(\frac{(MN + h_2)}{s_2} + \frac{(OQ + h_4)}{s_3}\right) - \left(\frac{(LM - h_1)}{s_1} + \frac{(NO - h_3)}{s_2}\right)\right]\frac{tYS}{100}$$

$$= \left(\frac{h_2}{s_2} + \frac{h_4}{s_3} + \frac{h_1}{s_1} + \frac{h_3}{s_2}\right)\frac{tYS}{100} \text{ sta-yds.,} \qquad (63)$$

since $\frac{MN}{s_2} + \frac{OQ}{s_3} = \frac{LM}{s_1} + \frac{NO}{s_2}$. H' is positive. Therefore raising

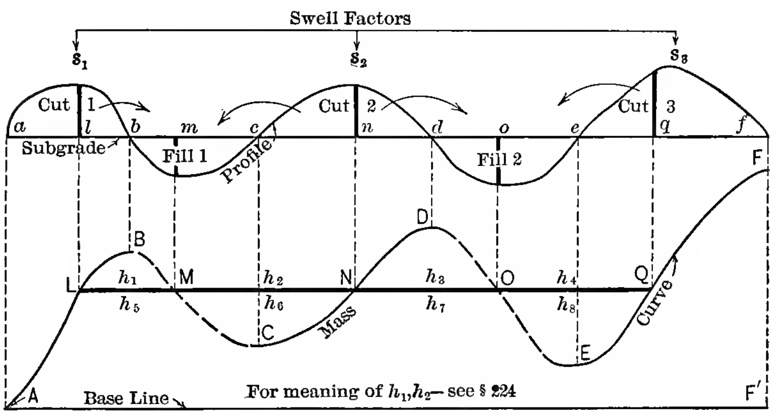


FIG. 66.

the balancing line above the position LQ increases the haul, and is not economical.

If we drop the balancing line a small distance t below the position LQ, the resulting net increase in the haul is

$$H'' = \left[\left(\frac{(LM + h_5)}{s_1} + \frac{(NO + h_7)}{s_2}\right) - \left(\frac{(MN - h_6)}{s_2} + \frac{(OQ - h_8)}{s_3}\right)\right]\frac{tYS}{100}$$

$$= \left(\frac{h_5}{s_1} + \frac{h_7}{s_2} + \frac{h_6}{s_2} + \frac{h_8}{s_3}\right)\frac{tYS}{100} \text{ sta-yds.,} \qquad (64)$$

since $\frac{LM}{s_1} + \frac{NO}{s_2} = \frac{MN}{s_2} + \frac{OQ}{s_3}$. So H'' is positive, and dropping the balancing line below the position LQ increases the haul.

We find that the haul is increased whether we raise or lower the balancing line from the position LQ, which means that LQ is the economic continuous balancing line.

In Sec. 215 it is shown that the continuous economic balancing line is not in every case *the* economic balancing line. Sometimes the form of the mass curve is such as to make it possible to reduce the haul resulting from the balancing line LQ by breaking that line at N. The principle brought out in Sec. 215 applies here.

225. When swell-factors are all equal. — If the swell-factors of any section of this chapter each be replaced by 1, the section will reduce to the corresponding section of the preceding chapter. For example, eq. 62 will then become eq. 51. Moreover, if the swell-factors of any section of this chapter be given a uniform value other than 1, the section will become about as simple as the corresponding section of the preceding chapter. With this change eq. 62, for example, will reduce to eq. 51.

226. Practical use of mass curve in planning distribution. — The work of planning distribution with mass curve based on equal swell-factors is little greater than with mass curve based on the common swell-factor 1 (Sec. 118). On the contrary, the additional work required with mass curve based on unequal swell-factors is considerable. However, it is not often that data pertaining to future swell and yardage is so reliable as to justify the adoption of unequal swell-factors in planning distribution. In the majority of cases the engineer will use the swell-factor 1 for all the cuts involved in one distribution problem; and in most of the cases in which the data will warrant deviating from the swell-factor 1, will be justified in adopting some other uniform swell-factor.

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